Analytical Skills - II

DEPEA516

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LOVELY PROFESSIONAL UNIVERSITY

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Unit 01: Time and Work

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Objectives

After studying this unit, you will be able to

- solve different types of time and work-related questions.
- Efficiency impact on work.
- What happened when number of people are increased to do a work.

Introduction

In this topic, we will learn about time, work, efficiency, and wages. Time and work problems deal with the simultaneous performance involving the efficiency of an individual or a group and the time taken by them to complete a piece of work. Work is the effort applied to produce a deliverable or accomplish a task.

A certain amount of time (T) is taken to complete a certain work (W). The number of units of work done per unit time is called the rate of work (R) or efficiency. Hence,

Work(W) = Rate (R) x Time (T)

Whenever some work is done, the total work itself can be taken as one unit.Hence, we assume the total work done as one unit in the problems we encounter to simplify the computations. In these cases, R = 1 / T or T = 1 / R. In other words, R and T are inversely proportional as **RT = W**, which is a fixedquantity.

1.1 Concept of Efficiency

Insuchquestions,therates atwhichsomeindividualscompleteaworkalonearegivenandyouare requiredtocalculatetherateatwhichtheycancompletetheworktogetherandviceversa.

Example -1 Ram is 50% more efficient than Shyam. If Shyam takes 15 days to complete a work. In how many days will the work be completed if they work together?

A) 10 days B) 8 days C) 6 days D) 7 days

Solution: As we know efficiency is inversely proportional to the time.so if we inverse the ratio of efficiency, we will get the ratio of time.

	Ram	:	Shyam
E	150	:	100
E	3	:	2
T	2	:	3

In the question, by comparison 3=15 (Shyam does the work in 15 days)

1= 15/3=5

2= 5*2= 10 days

Let the total work be LCM (10, 15) = 30 units

=> Ram's efficiency = 30/10 = 3 units /day

=> Shyam's efficiency = 30/15 = 2 units /day

Combined efficiency of Ram and Shyam = 3+2 = 5 units / day

=> In one day, Ram and Shyam working together can finish of 5 units of work, out of the given 30 units.

Therefore, time taken to complete total work = 30 / 5 = 6 days

1.2 Work

Work is directly proportional to the efficiency of a person and number of days taken by that person.

Example2: To complete a work, a person A takes 10 days and another person B takes 15 days. If they work together, in how much time will they complete the work?

A) 11 days B) 8 days C) 6 days D) 10 days

Solution:- Let the total work be LCM (10, 15) = 30 units

=> A's efficiency = 30/10 = 3 units /day

=> B's efficiency = 30/15 = 2 units /day

Combined efficiency of A and B = 3+2 = 5 units / day

=> In one day, A and B working together can finish of 5 units of work, out of the given 30 units.

Therefore, time taken to complete total work = 30 / 5 = 6 days

Example 3: A, B and C can do a piece of work in 10 days, 15 days and 30 days respectively. In how many days was the work completed if they work together?

A) 10 days B) 5 days C) 8 days D) 7 days

Solution:

Let the total work be LCM (10, 15, 30) = 30 units

=> A's efficiency = 30/10 = 3 units / day

=> B's efficiency = 30/15 = 2 units / day

=> C's efficiency = 30/30 = 1 units / day

Combined efficiency of A and B = 3+2+1 = 6 units / day

=> In one day, A, B and C working together can finish of 6 units of work, out of the given 30 units.

Therefore, time taken to complete total work = 30 / 6 = 5 days

Example 4: Two friends A and B working together can complete an assignment in 4 days. If A can do the assignment alone in 12 days, in how many days can B alone do the assignment?

A) 10 days B) 6 days C) 8 days D) 7 days

Solution: Let the total work be LCM (4, 12) = 12

=> A's efficiency = 12/12 = 1 unit / day

=> Combined efficiency of A and B = 12/4 = 3 units / day

Therefore, B's efficiency = Combined efficiency of A and B – A's efficiency = 2 units / day

So, time taken by B to complete the assignment alone = 12/2 = 6 days

Type 2: Leaving and Joining

Example 5: A and B can complete a piece of work in 10 and 15 days. they work together for 2 days and after That A leaves the work. In how many days will B complete the remaining work?

A) 10 days B) 6 days C) 8 days D) 7 days

Solution:

Let the total work be LCM (10, 15) = 30 units

=> A's efficiency = 30/10 = 3 units / day

=> B's efficiency = 30/15 = 2 units / day

Combined efficiency of A and B = 3+2 = 5 units / day

In 2 days, they will work= 5*2= 10 units

Reaming work= 30-10=20 units

B completes this work = 20/2=10 days

Type 3: Alternate Working

Example 6: A and B can complete a piece of work in 10 and 15 days. In how many days will the work be completed if they work on alternate days?

A) 10 days B) 12 days C) 8 days D) 7 days

Solution:

Let the total work be LCM (10, 15) = 30 units

=> A's efficiency = 30/10 = 3 units / day

=> B's efficiency = 30/15 = 2 units / day

In 2 days, they will work= 5 units

2 days= 5 units

Multiplying by 5 both side than

5*2 days= 5*5 units

10 days=25 units

On 11th day A will do= 3units

On 12th day B will do= 2 units

So work will be completed in 12 days.

Example 7: A, B and C can complete a piece of work in 10, 15 and 30 days respectively. If A is assisted by B and C on every third day than in how many days the work will be completed?

A) 10 days B) 11 days C) 8 days D) 7 days

Solution:

Let the total work be LCM (10, 15, 30) = 30 units

=> A's efficiency = 30/10 = 3 units / day

=> B's efficiency = 30/15 = 2 units / day

=> C's efficiency = 30/30 = 1 units / day

Combined efficiency of A and B = 3+2+1 = 6 units / day

According to question, A will work on first and second day and third day all A, B and C will work.

In 3 days= [A+A+(A+B+C)]

In 3 days =[3+3+6]= 12 units

Next 3 days= 12 units

On 7^{th} day A will do= 3 units

On 8^{th} day A will do =3 units

In 8 days work done= (12+12+3+3=30 units)

Example 8: If 2 men and 3 boys can do a piece of work in 10 days while 4 men can do the same in 10 days what is time taken by 3 men and 3 boys?

A) 4 days B) 6 days C) 8 days D) 5 days

Solution:

Given that

2 men and 3 boys can do a piece of work in 10 days

4 men can do the same in 10 days

As the work done is equal,

10(2M + 3B) = 10(4M)

20M + 30B = 40M

= 2M = 3B

= M/B= 3/2 ……(1)

(3M+3B)*X=4M*10

Putting M=3 and B=2 from equation 1

(3*3+3*2)*X= 4*3*10

After solving the equation X= 8 days

Example 9: If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days what is time taken by 15 men and 20 boys?

A) 4 days B) 6 days C) 7 days D) 5 days

Solution:

Given that

6 men and 8 boys can do a piece of work in 10 days

26 men and 48 boys can do the same in 2 days

As the work done is equal,

10(6M + 8B) = 2(26M + 48B)

60M + 80B = 52M + 96B

= M = 2B

= B = M/2 …… (1)

Now Put (1) in 15M + 20B

= 15M + 10M = 25M

Now, 6M + 8B in 10 days

= (6M + 4M) 10 = 100M

Then D(25M) = 100M

= D = 4 days.

1.3 Wages

The wages paid for any task has to be divided among the workers in the proportion to their contribution towards the completion of the task. In other words, the money earned by completing a piece of work has to be divided in the proportion of the work done by them.

If the workers have worked for the same number of days, the money can be divided in the ratio of their efficiencies. Efficiency is inversely proportional to the time taken to complete a task.

Wages questions are of three types:

Same efficiency and same number ofdays.

Different efficiency but same number ofdays.

Different efficiency and different number ofdays. Let's look at the questiontogether.

Example 10: A can complete a task in 10 days and B can complete the same task in 15 days. They start working together but A works for only 1 day. The remaining work is completed by B. If the total wage is 1000, then What is B's share?

Answer:

Now, the question says that A can complete a task in 10 days and B can dothe same in 15days

A = 10 days , B = 15 days

Remember the chocolate method? Total work assumed will be? LCM of (10 , 15) = 30

So efficiency, A= 3 chocolates /day and B= 2 chocolates /day

A works for 1 day which means that the work done by him will be: eating 3 chocolates

Work left = 30 - 3 = 27

Now eating 27 chocolates is the work completed by B Remember:

The wage is divided in the ratio of the work done. Ratio of work = 3:27 = 1:9

So the wage will be divided in the same ratio. Wage of A : Wage of B = 1:9So B's share = (9/10) x 1000

= Rs. 900

- If a person can do a piece of work in **'n'** days, then in one day, the person will do **'1/n'** work. Conversely, if the person does **'1/n'** work in one day, the person will require **'n'** days to finish thework.

- In questions where there is a comparison of work and efficiency, we usethe formula

  $\mathbf{M_1 D_1 H_1 E_1 / W_1 = M_2 D_2 H_2 E_2 / W_2}$, where

  M = Number of workers D = Number of days

  H = Number of working hours in a day E = Efficiency of workers

  W = Units of work

- In case we have more than one type of workers, then the formulamodifies to

$\mathbf{\sum(M_i\ E_i)\ D_1\ H_1 / W_1 = \sum(M_j\ E_j)\ D_2\ H_2 / W_2}$, where 'i' and 'j' may vary as per the number of workers.

- If a person A is 'n' times more efficient than person B,then

Ratio of work done by A and B in one day (Ratio of efficiencies) = **n : 1**

Ratio of time taken by A and B = **1 : n**

- Total work = No. of Days xEfficiency

- If a group of people are given salary for a job they do together, their individual salaries are in the ratio of their individual efficiencies if theywork for same number of days. Otherwise, salaries are divided in the ratio of units of work done.

In this topic, the way of attempting the questions is the deciding factor for getting accurate answer in less time. We shall try to cover all the types of questionsasked in this topic with detailed explanation of way of attemptingthem.

Example 11: A and B can do a piece of work in 12 and 20 days. A works for 3 days and leave the work than B works for 6 days and leave the work. The reaming work is done by C. if they receive 1200 Rs as total wage. Find out the amount received by C?

Solution:

Let the total work be LCM (12, 20) = 60 units

=> A's efficiency = 60/12= 5 units / day

=> B's efficiency = 60/20 = 3 units / day

Amount of work done by A= 5*3=15 units

Amount of work done by B= 3*6= 18 units

Amount of work done by C= [60-(15+18)]= 27 units

Now wage will divide in the ratio of work done.

Work done ratio of A:B:C=15:18:27= 5:6:9

Wages ratio of A:B:C= 5:6:9

Their individual wages will be= 5x, 6x, 9x

Total Wage=(5x+6x+9x)= 1200 => 20x=1200 => x=60

Total wage of C = 9x=9*60= 540 Rs

Example 12: 3 Men working 8 hours a day can make a toy in 2 days. how many hours a day must 4 men work to make the toy in 1 day?

Solution: M1*D1*H1=M2*D2*H1

3*2*8= 4*1*x

X= 12 hour

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts of Time & Work.
- We have learnt tricks to solve different types of Time & Work Problems
- We have learnt about the efficiency.
- Learnt to calculate how to equate work with respect to their efficiency.
- We have learnt the Wages concept.

Keywords

- Time & Work
- Efficiency
- Wages
- MDH/W formula

SelfAssessment

1. If A and B together can complete a piece of work in 15 days and B alone in 20 days, in how many days can A alone complete the work?

A. 60 days
B. 45 days
C. 40 days
D. 30 days

2. A work can be completed by P and Q in 12 days, Q and R in 15 days, R and P in 20 days. In how many days P alone can finish the work?

A. 10 days
B. 20 days
C. 30 days
D. 60 days

3. If 20 women can lay a road oflength 100m in 10 days. Then10 women can lay the sameroad of length 50m in

A. 20 days
B. 15 days
C. 5 days
D. 10 days

4. A and B together can do a pieceof work in 36 days, B and C together can do it in 24 days. Aand C together can do it in 18 days. The three working together can finish the work in

A. 8 days
B. 16 days
C. 30 days
D. 32 days

5. Ganesh, Ram and Sohan together can complete a work in 16days. If Ganesh and Ram together can complete the same workin 24 days, the number of days Sohan alone takes, to finish thework is

A. 40
B. 48
C. 32
D. 30

6. A and B can complete a work in15 days and 10 days respectively. They started doing thework together but after 2 days,B had to leave and A alone completed the remaining work. Thewhole work was completed in :

A. 10 days
B. 8 days
C. 12 days
D. 15 days

7. A man and a boy can complete awork together in 24 days. If forthe last six days man alone doesthe work then it is completed in26 days. How long the boy willtake to complete the work alone ?

A. 72 days
B. 20 days
C. 24 days
D. 36 days

8. A and B together can complete awork in 12 days. A alone can complete in 20 days. If B does thework only half a day daily, thenin how many days A and B together will complete the work ?

A. 10 days
B. 20 days
C. 11 days
D. 15 days

9. Ramesh and Rahman can do awork in 20 and 25 days respectively. After doing collectively for10 days at the work, they leavethe work due to illness andSuresh completes rest of the workin 3 days. How many daysSuresh alone can take to complete the whole work ?

A. 32 days
B. 28 days
C. 29 days

D. 30 days

10. A man, a woman and a boy cancomplete a work in 20 days, 30days and 60 days respectively.How many boys must assist 2men and 8 women so as to complete the work in 2 days ?

A. 8

B. 12

C. 4

D. 6

11. 3 men and 4 boys can completea piece of work in 12 days. 4men and 3 boys can do thesame work in 10 days. Then 2men and 3 boys can finish thework in number of days is

A. 17.5 days

B. 5(5/11) days

C. 8 days

D. 22 days

12. Twenty women can do a work insixteen days. Sixteen men cancomplete the same work in fifteen days. The ratio between thecapacity of a man and a woman is

A. 3 : 4

B. 4 : 3

C. 5 : 3

D. 5 : 7

13. A company employed 200workers to complete a certainwork in 150 days. If only one-fourth of the work has beendone in 50 days, then in orderto complete the whole work in time, the number of additional workers to be employed was

A. 100

B. 300

C. 600

D. 200

14. A is thrice as good a workman asB and therefore is able to finish ajob in 40 days less than B.Working together, they can do itin

A. 14 days

B. 13 days

C. 20 days

D. 15 days

15. 4 mat-weavers can weave 4 matsin 4 days. At the same rate howmany mats would be woven by8 mat-weavers in 8 days ?

A. 4

B. 8

C. 12

D. 16

Answers for SelfAssessment

l.	A	2.	C	3.	D	4.	B	5.	B
6.	C	7.	A	8.	D	9.	D	10.	A
11.	A	12.	B	13.	A	14.	D	15.	D

Review Questions

1.A and B can do a piece of workin 12 days, B and C in 8 daysand C and A in 6 days. How longwould B take to do the samework alone ?

(A) 24 days (B) 32 days(C) 40 days (D) 48 days

2.While working 7 hours a day, Aalone can complete a piece ofwork in 6 days and B alone in 8days. In what time would theycomplete it together, working 8hours a day ?

(A) 3 days (B) 4 days(C) 2.5 days (D) 3.6 days

3.15 men take 20 days to complete a job working 8 hours a day. Thenumber of hours a day should20 men take to complete the jobin 12 days

(A) 5 hours (B) 10 hours(C) 15 hours (D) 18 hours

4.A and B can do a piece of workin 15 days. B and C can do thesame work in 10 days and A andC can do the same in 12 days.Time taken by A, B and C together to do the job is

(A) 4 days (B) 9 days(C) 8 days (D) 5 days

5.A and B can do a piece of workin 18 days, B and C in 24 days,A and C in 36 days. Working togetherthey can do the work in

(A) 12 days (B) 13 days(C) 16 days (D) 26 days

6.If 10 men or 20 boys can make260 mats in 20 days, then howmany mats will be made by 8men and 4 boys in 20 days?

(A) 260 (B) 240(C) 280 (D) 520

7.4 men and 6 women can completea work in 8 days, while 3men and 7 women can completeit in 10 days. In how many days will 10 women complete it ?

(A) 50 days (B) 45 days(C) 40 days (D) 35 days

8.18 men or 36 boys working 6hours a day can plough a field in24 days. In how many days will24 men and 24 boys working 9hours a day plough the samefield ?

(A) 9 (B) 10(C) 6 (D) 8

9.A can cultivate 2/5 th of a land in6 days and B can cultivate1/3 rdof the same land in 10 days.

Working together A and B cancultivate4/5 th of the land in:

(A) 4 days (B) 5 days(C) 8 days (D) 10 days

10.A does half as much work as Bin three- fourth of the time. Iftogether they take 18 days tocomplete a work, how muchtime shall B take to do it alone?

(A) 30 days (B) 35 days(C) 40 days (D) 45 days

11.A can do a piece of work in 70days and B is 40% more efficientthan A. The number of days takenby B to do the same work is

(A) 40 days (B) 60 days(C) 50 days (D) 45 days

12.A is twice as good a workman asB and B is twice as good a workmanas C. If A and B can together

finish a piece of work in 4 days,then C can do it by himself in

(A) 6 days (B) 8 days(C) 24 days (D) 12 days

13.5 men and 2 women workingtogether can do four times asmuch work per hour as a man anda woman together. The work doneby a man and a woman should bein the ratio.

(A) 1 : 2 (B) 2 : 1(C) 1 : 3 (D) 4 : 1

14.39 persons can repair a road in12 days working 5 hours a day. In how many days will 30 personsworking 6 hours a day completethe work ?

(A) 10 days (B) 13 days(C) 14 days (D) 15 days

15.If 6 persons working 8 hours aday earn 8400 per week, then9 persons working 6 hours a daywill earn per week

(A) 8400 (B) 16800(C) 9450 (D) 16200

16.A daily-wage labourer was engaged for a certain number of days for 5,750; but being absent on some of those days he was paid only 5,000. What was his maximum possible daily wage?

(A) 125 (B) 250 (C) 375 (D) 500

17.A takes 10 days less than the time taken by B to finish a piece of work. If both A and B can do it in 12 days, then the time taken by B alone to finish the work is

(A) 30 days (B) 27 days (C) 20 days (D) 25 days

18. A works twice as fast as B. If B can complete a piece of work independently in 12 days, then what will be the number of days taken by A and B together to finsh the work?

(A) 4(B) 6 (C) 8 (D) 18

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R S Aggarwal, S Chand Publishing
2. Magical Book on Quicker Math's by M Tyra, Banking Service Chronicle

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 02: Pipes and Cistern

CONTENTS

Objectives

After studying this unit, you will be able to

- solve different types of pipes and cistern related questions.
- Efficiency impact on capacity of the tank.
- What happened when number of people are increased to do a work.

Introduction

This topic gives the relation between the time required to fill or empty the tank with the taps opened or closed. The problems of pipes and cisterns usually have two kinds of pipes, Inlet pipe and Outlet pipe / Leak. This chapter questions are similar to the time and work chapter. So the pattern of solving these questions are mostly similar to time and work questions.

2.1 Concept of Efficiency

Insuchquestions,therates atwhich tank or cistern is filled is called efficiency. It can be positive and negative depending on the inlet and outlet/leak pipe respectively.

2.2 Capacity of Tank

Capacity of the tank is the product of the efficiency of pipe and time taken by that pipe to fill the tank.

Inlet Pipe: Inlet is a pipe connected with a tank or cistern or reservoir. It is used to fill the tank.

- Outlet Pipe / Leak: Outlet is a pipe connected with a tank or cistern or reservoir. It is used to empty the tank.
- Time – Stands for time taken for filling or emptying.
- When a cistern is filled completely, then amount of work done (filling) = 1

Some Important Tricks

- If a pipe can fill a tank in x hours, part of the tank filled in 1 hour = 1/x

- If a pipe can empty a tank in y hours, part of the tank emptied in 1 hour = 1/y
- If a pipe can fill a tank in x hours and another pipe can fill it in y hours, then both the pipes together can fill the tank in [xy / x+y] hours.
- Suppose a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours. We can examine two cases here.

➢ If $x<y$

Net part filled in 1 hour = [1/x – 1/y]

➢ If $y<x$

Net part emptied in 1 hour = [1/y – 1/x]

- If Pipe A working alone takes 'a' hours more than A and B, and Pipe B working alone takes 'b' hours more than A and B together, then the number of hours taken by A and B, working together, to completely fill a tank is given by √ab

Example 1: Two pipes A and B can fill a tank separately in 12 and 16 hours respectively. If both of them are opened together when the tank is initially empty, how much time will it take to completely fill the tank?

Solution:- Part of tank filled by pipe A in one hour working alone = 1 / 12

Part of tank filled by pipe B in one hour working alone = 1 / 16

Part of tank filled by pipe A and pipe B in one hour working together

= (1 / 12) + (1 / 16) = 7 / 48

Therefore, time taken to completely fill the tank if both A and B work together

= 48 / 7 hours

Another Method

Let the capacity of tank be LCM (12, 16) = 48 units

Efficiency of pipe A = 48 / 12 = 4 units / hour

Efficiency of pipe B = 48 / 16 = 3 units / hour

Combined efficiency of pipes A and B = 7 units / hour

Therefore, time taken to completely fill the tank = 48 / 7 hours

Example 2: Three pipes A, B and C are connected to a tank. Out of the three, A and B are the inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours, B fills the tank in 12 hours and C empties the tank in 30 hours. If all three are opened simultaneously, how much time does it take to fill / empty the tank?

Solution: Let the capacity of tank be LCM (10, 12, 30) = 60 units

Efficiency of pipe A = 60 / 10 = 6 units / hour

Efficiency of pipe B = 60 / 12 = 5 units / hour

Efficiency of pipe C = – 60 / 30 = – 2 units / hour (Here, '-' represents outlet pipe)

Combined efficiency of pipes A, B and C = 6 + 5 – 2 = 9 units / hour

Therefore, time taken to completely fill the tank = 60 / 9 = 6 hours 40 minutes

Example 3: A cistern has two pipes. Both working together can fill the cistern in 12 minutes. First pipe is 10 minutes faster than the second pipe. How much time would it take to fill the cistern if only second pipe is used?

Solution: Let the time taken by first pipe working alone be 't' minutes.

Time taken by second pipe working alone = t + 10 minutes.

Part of tank filled by pipe A in one hour working alone = 1 / t

Part of tank filled by pipe B in one hour working alone = 1 / (t + 10)

Part of tank filled by pipe A and B in one hour working together

= (1 / t) + (1 / t+10) = (2t + 10) / [t x (t + 10)]

But we are given that it takes 12 minutes to completely fill the cistern if both pipes are working together.

(2t + 10) / [t x (t + 10)] = 1 / 12

t x (t + 10) / (2t + 10) = 12

$t^2 + 10t = 24t + 120$

$t^2 – 14t – 120 = 0$

(t – 20) (t + 6) = 0

t = 20 minutes (Time cannot be negative)

Therefore, time taken by second pipe working alone = 20 + 10 = 30 minutes

Example 4: Three pipes A, B and C are connected to a tank. Out of the three, A and B are the inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours and B fills the tank in 30 hours. If all three are opened simultaneously, it takes 30 minutes extra than if only A and B are opened. How much time does it take to empty the tank if only C is opened?

Solution: Let the capacity of tank be LCM (10, 30) = 30 units

Efficiency of pipe A = 30 / 10 = 3 units / hour

Efficiency of pipe B = 30 / 30 = 1 units / hour

Combined efficiency of pipes A and B = 4 units/hour

Therefore, time taken to completely fill the tank if only A and B are opened

= 30 / 4 = 7 hours 30 minutes

Time taken to completely fill the tank if all pipes are opened

= 7 hours 30 minutes + 30 minutes = 8 hours

Combined efficiency of all pipes = 30 / 8 = 3.75 units / hour

Now, efficiency of pipe C = Combined efficiency of all three pipes – Combined efficiency of pipes A and B

Therefore, efficiency of pipe C = 4 – 3.75 = 0.25 units / hour

Thus, time taken to empty the tank if only C is opened = 30 / 0.25 = 120 hours

Example 5: Time required by two pipes A and B working separately to fill a tank is 36 seconds and 45 seconds respectively. Another pipe C can empty the tank in 30 seconds. Initially, A and B are opened and after 7 seconds, C is also opened. In how much more time the tank would be completely filled?

Solution: Let the capacity of the tank be LCM (36, 45, 30) = 180 units

Efficiency of pipe A = 180 / 36 = 5 units / second

Efficiency of pipe B = 180 / 45 = 4 units / second

Efficiency of pipe C = – 180 / 30 = – 6 units / second

Now, for the first 7 seconds, A and B were open.

Combined efficiency of A and B = 5 + 4 = 9 units / second

Part of the tank filled in 7 seconds = 7 x 9 = 63 units

Part of tank empty = 180 – 63 = 117 units

Now, all pipes are opened.

Combined efficiency of all pipes = 5 + 4 – 6 = 3 units / second

Therefore, more time required = 117 / 3 = 39 seconds

Example 6: Two pipes A and B can fill a tank in 20 hours and 30 hours respectively. If both the pipes are opened simultaneously, find after how much time should pipe B be closed so that the tank is full in 18 hours?

Solution: Let the capacity of the tank be LCM (20, 30) = 60 units

Efficiency of pipe A = 60 / 20 = 3 units / hour

Efficiency of pipe B = 60 / 30 = 2 units / hour

Combined efficiency of pipes A and B = 5 units / hour

Let both A and B be opened for 'n' hours and then, B be closed and only A be opened for the remaining '18 – n' hours.

5n + 3 x (18 – n) = 60

2n + 54 = 60

2n = 6

n = 3

Therefore, B should be closed after 3 hours.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts of Pipes&Cistern.
- We have learnt tricks to solve different types of Pipes and Cistern Problems
- We have learnt about the positive and negative efficiency.
- We have learnt how to find the capacity of cistern.

Keywords

- Pipes and Cistern
- Positive efficiency and Negative efficiency
- Inlet Pipe
- Outlet pipe
- Capacity of tank/cistern

SelfAssessment

1. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:

A. 113

B. 114

C. 115

D. 116

2. Pipe A can fill a tank in 8 hours, pipe B in 4 hours and pipe C in 24 hours. If all the pipes are open, in how many hours will the tank be filled?

A. 2.4 hr

B. 3 hr

C. 4 hr

D. 4.2 hr

3. Two pipes A and B can fill a tank in 2 and 6 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

A. 3 min

B. 2.5 min

C. 2 min

D. 1.5 min

4. Two outlet pipes A and B are connected to a full tank. Pipe A alone can empty the tank in 10 minutes and pipe B alone can empty the tank in 30 minutes. If both are opened together, how much time will it take to empty the tank completely?

A. 7 minutes

B. 7 minutes 30 seconds

C. 6 minutes

D. 6 minutes 3 seconds

5. Three pipes A, B and C were opened to fill a cistern. Working alone, A, B and C require 12, 15 and 20 mins respectively. Another pipe D, which is a waste pipe, can empty the filled tank in 30 mins working alone. What is the total time (in mins) taken to fill the cistern if all the pipes are simultaneously opened?

A. 5

B. 6

C. 7

D. 8

6. Pipes A and B can fill a tank in 8 and 24 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:

A. 18 hr

B. 6 hr

C. 24 hr

D. 12 hr

7. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

A. 6 min.to empty

B. 6 min.to fill

C. 9 min.to empty

D. 9 min.to fill

8. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes, and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of the solution R in the liquid in the tank after 3 minutes?

A. 5/11

B. 6/11

C. 8/11

D. 7/11

9. 13 buckets of water fill a tank when the capacity of each bucket is 51 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 17 litres?

A. 33

B. 29

C. 39

D. 42

10. Two pipes A and B can separately fill a cistern in 40 minutes and 30 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 20 minutes. In how much time, the third pipe alone can empty the cistern?

A. 120 min

B. 100 min

C. 140 min

D. 80 min

11. Two taps A and B can fill a tank in 5 hours and 20 hours respectively. If both the taps are open then due to a leakage, it took 40 minutes more to fill the tank. If the tank is full, how long will it take for the leakage alone to empty the tank?

A. 28 hr

B. 16 hr

C. 22 hr

D. 32 hr

12. A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 liters a minute. When the tank is full, the inlet is opened and due to the leak, the tank is empty in 24 hours. How many liters does the tank hold?

A. 4010 litre

B. 2220 litre

C. 1920 litre

D. 2020 litre

13. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in:

A. 20/3 hours

B. 6 hours

C. 15/2 hours

D. 7 hours

14. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time; A and B fill it together for the other half?

A. 15 min

B. 20 min

C. 27.5 min

D. 30 min

15. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is:

A. 10

B. 12

C. 14

D. 16

Answers forSelfAssessment

l.	C	2.	A	3.	D	4.	B	5.	B
6.	D	7.	A	8.	C	9.	B	10.	C
11.	A	12.	B	13.	B	14.	C	15.	A

Review Questions

1. A water tank is two-fifth full. Pipe A can fill a tank in 12 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

A. 2.8 min

B. 4.2 min

C. 4.8 min

D. 5.6 min

2. One pipe can fill a tank 6 times as fast as another pipe. If together the two pipes can fill the tank in 22 minutes, then the slower pipe alone will be able to fill the tank in:

A. 164 min

B. 154 min

C. 134 min

D. 144 min

3. Two pipes A and B can fill a tank is 8 minutes and 14 minutes respectively. If both the pipes are opened simultaneously, and the pipe A is closed after 3 minutes, then how much more time will it take to fill the tank by pipe B?

A. 6 min 15 sec

B. 5 min 45 sec

C. 5 min 15 sec

D. 6 min 30 sec

4. Two pipes A and B can fill a tank in 9 hours and 3 hours respectively. If they are opened on alternate hours and if pipe A is opened first, in how many hours will the tank be full?

A. 4 hr

B. 6 hr

C. 2 hr

D. 5 hr

5. A tap can fill a tank in 4 hours. After half the tank is filled, two more similar taps are opened. What is the total time taken to fill the tank completely?

A. 1 hr 20 min

B. 4 hr C.

3 hr D.

2 hr 40 min

6. A tap can fill a tank in 4 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?

A. 3 hr

B. 1 hr 30 min

C. 2 hr 30 min

D. 2 hr

7. One pipe can fill a tank four times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in:

A. 180 min

B. 144 min.

C. 126 min

D. 114 min

7. A tank is filled in 10 hours by three pipes A, B and C. Pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

A. 70 hours

B. 30 hours

C. 35 hours

D. 50 hours

8. Working alone, two pipes A and B require 9 hours and 6.25 hours more respectively to fill a pool than if they were working together. Find the total time taken to fill the pool if both were working together.

A. 6

B. 6.5

C. 7

D. 7.5

9. Two pipes A and B can fill a tank in 15 minutes and 40 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

A. 10 min 10 sec

B. 25 min 20 sec

C. 29 min 20 sec

D. 20 min 10 sec

10. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately?

A. 6 hours

B. 2 hours

C. 4 hours

D. 3 hours

11. Two pipes A and B can fill a cistern in 37 1/2 minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if pipe B is turned off after?

A. 5 min

B. 9 min

C. 10 min

D. 15 min

12. A pump can fill a tank with water in 2 hours. Because of a leak, it took 8/3 hours to fill the tank. The leak can drain all the water of the tank in?

A. 6 hours

B. 8 hours

C. 9 hours

D. 10 hours

13. Three pipes A, B and C was opened to fill a cistern. Working alone A, B and C require 12, 15 and 20 minutes respectively. After 4 minutes of working together, A got blocked and after another 1 minute, B also got blocked. C continued to work till the end and the cistern got completely filled. What is the total time taken to fill the cistern?

A. 6 minutes

B. 6 minutes 15 seconds

C. 6 minutes 40 seconds

D. 6 minutes 50 seconds

14. Three pipes A, B and C are connected to a tank. Working alone, they require 10 hours, 20 hours and 30 hours respectively. After some time, A is closed and after another 2 hours, B is also closed. C works for another 14 hours so that the tank gets filled completely. Find the time (in hours) after which pipe A was closed.

A. 2

B. 1.5

C. 1

D. 3

15. Two pipes A and B are connected to drain out a water tank. A alone can drain out the tank in 20 hours and B can drain 20 liters per hour. Find the capacity of the water tank given that working together they require 12 hours to completely drain out the tank.

A. 600

B. 400

C. 800

D. 700

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R S Aggarwal, S Chand Publishing
2. Magical Book on Quicker Math's by M Tyra, Banking Service Chronicle

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 03: Time, Speed and Distance

CONTENTS

Objectives

After studying this unit, you will be able to

- Understand the key concept of Time speed and distance
- Understand the relationship among Time speed and distance
- Understand tips and tricks to solve different type of problems
- Understand method to Finding Average speed
- Understand Logic to Finding Relative speed

Introduction

The basic concept of Time speed and Distance is the relationship among three variables. In this, we are going to cover the key concept of Time speed and Distance along with different types of problems and tips and tricks. Problems related to Time speed and Distance include various categories like straight line, relative motion etc. Therefore, candidates should try to learn the Inter-relationship amongst the factors of speed, distance, and time.

Distance(D): The numerical description of **"How far two objects"** is called distance. It may also refer to the length between two points that may be present or may get created.

Time(T): It is a component to compare the **"durations of events"** and the **"intervals"** between them and to quantify the motions of objects.

Speed(S): It refers to the rate at which a particular distance is covered by an object in motion.

3.1 Basic Relationship

Time Speed and Distance are the three variables that represent the mathematical model of motion as

- **Distance = Speed*Time**

In mathematical format:

The formula for Speed calculation is:

- **Speed = Distance/Time**

The formula for Time calculation is:

- **Time = Distance/Speed**

3.2 Units and Their Conversion

Units of Time Speed and Distance

Each of Time Speed and Distance can be represented in different Units:

- Time can be expressed in terms of Seconds(s), minutes(min) and hour(hr.)
- Distance can be expressed in terms of meter(m), kilometer(km), miles etc.
- Speed is commonly expressed in m/s, km/hr.

For example, if Distance is given in km and Time is given in hr., then as per the formula:

Speed = Distance/Time. The unit of Speed will become km/hr.

Time speed and Distance Conversion

Another important concept is the conversion of Time speed and distance into various Units as discussed below:

- 1 km = 1000 m
- 1 mile = 1.609 km
- 1 hr = 60 min
- 1 min = 60 s
- 1 hr = 3600 s
- To convert a given data from **km/hr to m/s,** we have to multiply by 5/18.

  As. 1 km/h = 1000m / 3600s = 5m / 18s

  So,

  1 km/h = 5/18 m/s
- To convert a given data from **m/s to km/hr,** we have to multiply by 18/5.

  As. 1 m/s = 18/5 km/hr

  So,

  1 m/s = 18/5km/hr

In terms of formula, we can list it as:

- X km/hr = X*5/18 m/s
- X m/s = X*18/5 km/hr

3.3 Proportionality

From the relation D= S*T, we find that when

(i) Distance is constant, Speed and Time are inversely proportional.

$S \propto 1/T$ (when distance is constant)

For the same distance, when the speed increases, the time taken decreases and vice versa.

Let's assume a person travels at 'a' km/h from his home to reach his office and then decides to travel at 'b' km/h to return to his home.

The ratio of the speeds is given by,

S1:S2 = a: b

With distance constant, we know that the time taken is inversely proportional to the speed.

Thus, the ratio of the time is given by the inverse ratio of the speeds,

T1: T2 = 1/a: 1/b = b: a

Corollary: When two or more persons travel the same distance, the ratio of the time taken by them is given by the inverse ratio of their speeds and vice versa.

Example:Abhay takes 3 hours to travel from City A to City B at his normal speed. However, he can travel the same distance in 2 hours by travelling at 'x' km/h. Find the normal speed of Abhay if the difference between the speeds is 20 km/h.

Solution:

Here, distance is constant, and we can find the ratio of the speeds by inversing the ratio of the time taken, T1:T2 = 3:2

Hence, the ratio of the speeds is given by, S1:S2 = 2:3

From the question, we know that the difference between the speeds is 20 km/h.

So, from the ratio S1:S2, we get = $2x$:$3x$,

where the difference is x = 20 km/h

- Therefore, by substituting the value of x we find that the speeds S1, S2 are 40 km/h and 60 km/h respectively.
- Hence, the normal speed of Abhay is 40 km/h.

(ii) **Time is constant, Speed and Distance are directly proportional**

S ∝ D (when time is constant)

For a given time, the distance travelled by a particle increases with the increase in its speed.

Let's assume that a person travels 'a' km/h for a time 't' and then travels at 'b' km/h for the same time.

The ratio of the speeds is given by S1:S2 = a: b.

Since the distance travelled is proportional to the speed, the ratio of the distances travelled remains the same as that of the speeds. i.e., D1:D2 = a: b

Example:Two cars start from A and B travel towards each other at speeds of 50 km/h and 60 km/h respectively. At the time of their meeting, the second car has travelled 12 kms more than the first. Find the distance between A and B.

Solution:

Starting at the same time but from different points, the time taken by the cars to reach the meeting point are equal.

Hence, it can be inferred that the time is constant for both the cars to reach the meeting point.

The ratio of the speeds of car A to car B is S1:S2 =50:60= 5:6

The ratio of the distance travelled by them is D1:D2 = 5:6

From the question, we find that the car B has travelled 12 km more than the car A which is the difference between the distances travelled by the cars as reflected in the ratio $5x$:$6x$.

So, $5x \sim 6x = x = 12$

The total distance i.e., the distance between A and B is $5x + 6x = 11x = 11 \times 12 = 132$ km

Therefore, the distance between A and B is 132 km

(iii) **Speed is constant, Time and Distance are directly proportional**

T ∝ D (when S is constant)

Speed is a derived variable from Distance and Time.

So, in the relation

S = D/T,

if D increases, say 5D, then the time will also become 5T in order to maintain the L.H.S. of the relation when speed is constant.

3.4 Average Speed

Average speed can be defined as the total speed of a moving particle covering a distance in a particular time. Though the definition seems familiar and same as with that of a speed, the idea is conceived as follows.

- When a particle moves uniformly at a certain speed travelling a certain distance to reach a certain point, the average/total speed is the uniform speed itself.
- When a particle totally travels a certain distance in a total time and may not move at a uniform speed throughout, the average/total speed is calculated as the total distance travelled in a total time.

Thus, generalizing the above, we get

Average Speed = Total Distance covered/Total time taken

Or Average Speed = (d1+d2+d3+……dn)/ (t1+t2+t3+…tn)

Example:A man travelled 12 km at the speed of 4km/h and further 10 km at the speed of 5 km/h. what was his average speed?

Solution:

Total distance travelled= 12 +10 = 22 km

Total time taken = time taken at the speed of 14 km/h + time taken at the speed of 5 km/h

= 12/4 + 10/5

= 5 hr

Average Speed = Total distance travelled/Total Time taken

= 22/5 km/h

(i) When Distance is constant:

When the travelled distance is constant and two speed is given then:

Average Speed= 2s1s2/(s1+s2)

Where s1 and s2 are two speeds at which the corresponding distance has been reached.

Example:If the car had gone from A to B at 20 kmph and returned from B to A at 30 kmph, what would be the average speed?

Solution:

Let d be the A – B distance,

Then the average speed = d + d/ t1 + t2

= 2d / {(d/s1) + (d/s2)}

= 2* s1*s2 / (s1 + s2)

= 2 * 20 * 30 / (20 + 30)

= 24 km/h

(ii) **When Time is constant:**

When the time taken is constant Average speed is calculated by the formula:

Average Speed = (s1+s2)/2

Where s1 and s2 are two speeds at which we covered the distance for the identical time.

3.5 Relative Speed

Relative Speed is defined as the speed of one particle with respect to the other.

Let's say two boys B1 and B2 start running from the same point at the same time. The speeds of B1, B2 is 4 m/s and 7 m/s respectively.

Case 1: When they run in the same direction

After a second, B2 leads B1 by 3 m. The speed of B2 with respect to B1 is 3 m/s. As they continue running, the gap between them increases at the rate of 3 m/s.

Here, the relative speed is the difference between the speeds 7 – 4 = 3 m/s as it is perceived as the relative displacement in unit time.

For the same scenario, if B1 were 12 m ahead of B2 before they started to run, then the gap between them would initially decrease and then increase at the rate of 3 m/s.

Case 2: When they run in opposite direction

After a second, the gap between B1 and B2 is 11 m. The speed of B2 with respect to B1 is 11 m/s. As they continue running, the gap between them increases at the rate of 11 m/s.

Here, the relative speed is the sum of speeds 7 + 4 = 11 m/s as it is perceived as the relative displacement in unit time.

For the same scenario, if there was an initial gap of 12 m between B1 and B2, facing each other, before they started to run, then the gap between them would initially decrease and then increase at the rate of 11 m/s.

Corollary: The gap between two moving bodies increases/ decreases at the rate of their relative speed.

- When the bodies move in the same direction, the **relative speed** = difference between the speeds = **S1 ~ S2**
- When the bodies move in the opposite directions, the **relative speed** = sum of the speeds = **S1 + S2**

Steps involved in applying relative speed to solve problems

- Determine the initial distance between the particles.

If both particles move simultaneously, the distance between them is the initial distance and if they do not start at the same time, calculate the distance between them when the late starter begins to move and take that as the initial distance.

- Evaluate the relative speed.
- Calculate the time taken for the two particles to meet or cross each other as the ratio of initial distance to the relative speed.

Example:If two persons are moving at 10 km/h and 20 km/h in opposite directions, then their relative speed would be 10 + 20 = 30 km / h. Similarly, if they were moving in the same direction, their relative speed would be 20 – 10 = 10 km / h.

Example:Two stations B and M are 465 km distant. A train starts from B towards M at 10 AM with the speed 65 km / hr. Another train leaves from M towards B at 11 AM with the speed 35 km / hr. Find the time when both the trains meet.

Solution:

The train leaving from B leaves an hour early than the train that leaves from M.

Distance covered by train leaving from B = 65 km / hr x 1 hr = 65 km

Distance left = 465 – 65 = 400 km

Now, the train from M also gets moving and both are moving towards each other.

Applying the formula for relative speed,

Relative speed = 65 + 35 = 100 km / hr

Time required by the trains to meet = 400 km / 100 km / hr = 4 hours

Thus, the trains meet at 4 hours after 11 AM, i.e., 3 PM.

3.6 Different Type of Problems

Example:A man covers 30 km distance with speed of 10 kmph and return with 15 kmph. Find average speed.

Solution: **Average Speed**= 2s1s2/(s1+s2)

= 2 * 10 * 15 / (10 + 15)

= 12 km/h

Example:A man covers 60 km distance with speed of 10, 12 and 15 kmph on three successive days. Find average speed.

Solution: Total distance travelled= 60 +60+60 = 180 km

Total time taken = 60/10 + 60/12 + 60/15

= 15 hr

Average Speed = Total distance travelled/Total Time taken

180/15= 12 km/hr

Example:A boy goes to school with speed of 3 kmph and return with 2 kmph. If he takes 5 hrs in all. Find distance between home and school.

Solution:

Speed Ratio = 3:2

Time Ratio = 2: 3

Total = 5 part = 5hr

So, 1 part = 1hr

Hence time taken = 2hr&3hr

Distance = 3 km in 2hr or 2 km in 3hr = 6 km.

Example:A boy walks at 5 kmph and reaches his school 10 min late. If speed has been 6 kmph, he will reach 15 min early. Find distance between home and school.

Solution:

Speed Ratio = 5: 6

Time Ratio = 6: 5

Difference = 1 part = 25 min

Hence time taken = 150 min&125 min

Distance covered = 5 km in 150 min= 12.5 km

Example:If train runs at 40 kmph, it reaches its destination late by 11 min but if it runs at 50 kmph, it is late by 5 min. Find the correct time for train to complete the journey.

Solution:

Speed Ratio = 4: 5

Time Ratio = 5: 4

Difference = 1 part = 6 min

Hence time taken = 30 min&24 min

Distance covered = 40 km in 30 min = 20 km

Correct time = 30 – 11 or 24 – 5 = 19 min

Example:Walking 5/6 of usual speed a man is late by 10 min. Find the usual time taken by him to cover the distance.

Solution:

Speed Ratio = 1: 5/6 = 6: 5

Time Ratio = 5/6: 1= 5: 6

Difference = 1 part = 10 min

Hence Usual time = 5*10 = 50 min

Example:Two bikes start from A and B towards each other with 16 km/h and 21 km/h resp. When they meet it is found that second bike has travelled 60 km more. Find distance between A and B.

Solution:

Speed Ratio = 16: 21

Distance Ratio = 16: 21

Difference= 5 part= 60 km

So, 1 part = 12 km

Hence Distance between A and B = (16+21) *12= 444 km

Example:The distance between two cities A and B is 330 km. A train starts from A at 8 a.m. and travels towards B at 60 km/hr. Another train starts from B at 9 a.m. and travels towards A at 75 km/hr. At what time do trains meet?

Solution:

The train leaving from A leaves an hour early than the train that leaves from B.

Distance covered by train leaving from A = 60 km / hr x 1 hr = 60 km

Distance left = 330 – 60 = 270 km

Now, the train from B also gets moving and both are moving towards each other.

Applying the formula for relative speed,

Relative speed = 60 + 75 = 135 km / hr

Time required by the trains to meet = 270 km / 135 km / hr = 2 hours

Thus, the trains meet at 2 hours after 9 AM, i.e., 11AM.

Example:A train travelling with 25km/h leaves Delhi at 9 am and another train travelling with 35km/h leaves Delhi at 2 pm in the same direction. How far from Delhi will these together?

Solution: The train starting at 9 a.m. has travelled by 2 p.m. a distance of (5×25) km. i.e., 125 km.

Relative speed of two trains = (35−25); i.e., 10 km. per hour.

The second train shall meet the first train in 125/10; i.e., 12.5 hrs.

Distance from Delhi=12.5×35 km=437.5 km.

Example:A police officer chases a thief. Their respective speeds are 8 and 6 km/hr. If the police officer started 10 min late, at what distance he will catch the thief?

Solution:

Distance covered by the thief in 10 minutes =6×10/60=1km

The relative speed = 8 -6 = 2 km/h.

Time taken by police to cover 1 km. = 1/2 hr.

The speed of the police = 8 km.

In 1/2 hr. the police will cover 1/2*8= 4 km.

The police will catch the thief at 4 km. distance.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts of Time Speed and Distance.
- We have learnt tricks to solve different types of Time Speed and Distance Problems
- We have learnt about conversion of Units of Time Speed and Distance
- Learnt to calculate Average and Relative Speed.
- We have learnt the relationship among Time Speed and Distance

Keywords

- Units of Time Speed and Distance
- Conversion of Units
- Proportionality
- Average Speed
- Relative Speed

Self Assessment

1. A boy goes to school with speed of 6 kmph and return with 3 kmph. If he takes 5 hrs in all. Find distance between home and school.
   A. 5 km
   B. 6 km
   C. 10 km
   D. 12 km
2. Walking at 4 km/h a clerk reaches his office 5 min late. If he walks at 5 km/h he reaches 2 and half min earlier. What's the distance?
   A. 15 km
   B. 10 km
   C. 25 km
   D. 2.5 km

3. To cover same distance, time taken by two trains is in ratio 5: 7. If the second train runs 400 km in 4 hours, what is the speed of the first train?
   A. 85 km/hr.
   B. 140 km/hr.
   C. 90 km/hr.
   D. 120 km/hr.

4. By walking at 3/7th of his usual speed, a man reaches office 20 minutes later than usual time. What is his usual travel time?
   A. 10 min
   B. 15 min
   C. 20 min
   D. 25 min

5. A train leaves Delhi at 6 AM and reaches Agra at 10 AM. Another train leaves Agra at 8 AM and reaches Delhi at 11:30 AM. At what time the trains will cross each other?
   A. 8: 32 AM
   B. 8: 48 AM
   C. 8: 52 AM
   D. 8: 56 AM

6. Without any stoppage a person travels a certain distance at an average speed of 80 km/h and with stoppage he covers the same distance at an average speed of 60 km/h. How many minutes per hour does he stops?
   A. 45 min
   B. 30 min
   C. 20 min
   D. 15 min

7. Distance between two stations A and B is 208 km. A train starts from station A at 10 AM with 30 km/h and another starts from B at 1:20 noon with 24 km/h. When will the train meet and how far from station A?
   A. 2:20 PM, 120 km
   B. 3:20 PM, 160 km
   C. 2:20 PM, 160 km
   D. 3:20 PM, 120 km

8. A thief is noticed by a policeman from 200 m. the thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km/hr and 11 km/hr respectively. What is the distance between them after 6 minutes?
   A. 100 m
   B. 190 m
   C. 200 m
   D. 150 m

9. A man covers a certain distance by car driving at 30kmph and he returns to the starting point with a speed 40kmph. Find his average speed for the whole journey.
   A. 35 km/hr.
   B. 34.3 km/hr.
   C. 36 km/hr.
   D. None

10. A man covers 1/4 of his journey at 20 km/hr. and the remaining at 30 km/hr. He takes 15 hours in total journey. The distance total journey is?
    A. 400 km
    B. 300 km
    C. 390 km
    D. None

11. If a person walks at 14 km/hr. instead of 10 km/hr., he would have walked 20 km more. The actual distance travelled by him is:
    A. 50 km
    B. 56 km
    C. 70 km
    D. 80 km

12. In a fight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr. and the time of flight increased by 30 minutes. Find the duration of flight.
    A. 1 hr.
    B. 2 hr.
    C. 3 hr.
    D. 4 hr.

13. A Man travelled a distance of 61 km in 9 hours. He travelled partly on foot at 4 km/hr and partly on bicycle at 9 km/hr. What is the distance travelled on foot?
    A. 16 km
    B. 14 km
    C. 12 km
    D. 10 km

14. An express train travelled at an average speed of 100 km/hr., stopping for 3 minutes after every 75 km. How much time does the Train take to travel between two places which are 600 km apart?
    A. 6 hrs 21 min
    B. 6 hrs 24 min
    C. 6 hrs 27 min
    D. 6 hrs 30 min

15. A train after running 100 km meet with an accident and then run at 3/5th of its former speed and reaches the destination late by 48 min. If the accident had happened 30 km further, it will be late by 24 min. Find speed of train.

A. 125 km/hr

B. 150 km/hr

C. 100 km/hr

D. 50 km/hr

Answers for SelfAssessment

l.	C	2.	D	3.	B	4.	B	5.	D
6.	D	7.	B	8.	A	9.	B	10.	A
11.	A	12.	A	13.	A	14.	A	15.	D

Review Questions

1. 3 person A, B and C covers a distance at 10 km/hr. ,12 km/hr. and 15 km/hr. Find the average speed.
2. A man covers 1/4 of his journey at 20 km/hr. and the remaining at 30 km/hr. He takes 15 hours in total journey. The distance total journey is?
3. If a boy walks from his house at 4 km/hr. he reaches school 10 min early, if he walks at 3 km/hr. he reaches 10 min late. What is the distance from his house to school?
4. Walking at 7/8 of his usual speed a man late his office 15 min, find his usual time.
5. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R S Aggarwal, S Chand Publishing
2. Magical Book on Quicker Math's by M Tyra, Banking Service Chronicle

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

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Unit 04: Problem on Trains, Boats and Streams and Races

CONTENTS

Objectives

After studying this unit, you will be able to

- Understand the key concept of Problem on Trains
- Understand the concept of Relative speed
- Understand the concept of Boats and Streams and races
- Understand tips and tricks to solve different type of problems

Introduction

In this Chapter we will see the concepts of Problem on Trains, Boats and Streams and races. Similar to the concept of speed, distance and time, train problems are specifically based on evaluating the speed, distance covered, and time is taken by a train under different conditions.

Distance(D): The numerical description of **"How far two objects"** is called distance. It may also refer to the length between two points that may be present or may get created.

Time(T): It is a component to compare the **"durations of events"** and the **"intervals"** between them and to quantify the motions of objects.

Speed(S): It refers to the rate at which a particular distance is covered by an object in motion.

4.1 Trains- Basic Concept

Similar to the concept of speed, distance and time, train problems are specifically based on evaluating the speed, distance covered, and time is taken by a train under different conditions.

4.2 Important Formula

Distance = Speed * Time

- Speed of the Train = Total distance covered by the train / Time taken
- Relative Speed:
- When the bodies move in the same direction, the **relative speed** = difference between the speeds = **S1 ~ S2**
- When the bodies move in the opposite directions, the **relative speed** = sum of the speeds = **S1 + S2**
- If the length of two trains is given, say a and b, and the trains are moving in **opposite directions** with speeds of x and y respectively, then the **time taken by trains to cross each other = {(a+b) / (x+y)}**
- If the length of two trains is given, say a and b, and they are moving in the **same direction**, with speeds x and y respectively, then the **time is taken to cross each other = {(a+b) / (x-y)}.**
- When the **starting time of two trains is the same from x and y towards each other** and after crossing each other, they took t1 and t2 time in reaching y and x respectively, then the **ratio between the speed of two trains = √t2: √t1**

4.3 Type of Questions on Train Problems

- **Time Taken by Train to Cross a Pole or a telephone post**

Let the length of train is 'L' and its speed is 's'

Hence, Time to cross the pole is**: L/s**

- **Time Taken by Train to Cross a Stationary Man standing on the platform**

Let the length of train is 'L' and its speed is 's'

Hence, Time to cross the Man is**: L/s**

- **Time Taken by Train to Cross a moving Man**

Let the length of train is 'L' and its speed is 's'

Case I: If a Man is moving in the same direction of train with the speed of x

Relative speed: (s-x); s>x

Hence, Time to cross the Man is: **L/(s-x)**

Case II: If a Man is moving in the opposite direction of train with the speed of x

Relative speed: (s+x)

Hence, Time to cross the Man is: **L/(s-x)**

- **Time Taken by Train to Cross a Platform or a Bridge or a Canal etc.**

Let the length of train is 'L' and its speed is 's'

If the length of a platform is 'a', then Time to cross**: (L+a)/s**

If the length of a Bridge is 'b', then Time to cross**: (L+b)/s**

If the length of a Canal is 'c', then Time to cross**: (L+c)/s**

- **Time Taken by 2 trains to cross each other**

Let the length of Trains are 'a' and 'b' and their speeds are 'x' and 'y' respectively.

Case I: If trains are moving in the Opposite direction

Time to cross each other: **{(a+b) / (x+y)}**

Case II: If trains are moving in the Opposite direction

Time to cross each other: **{(a+b) / (x~y)}**

Example:A train is running at 36 km/h. If it crosses a pole in 25 s, its length is:

Solution:

Speed of train = 36 kmph

= 36*5/18= 10 m/s

∴ Length of train

= Speed × time

= 10 × 25 = 250 m

Example:A train 50 m long passes a platform 100 m long in 10 sec. The speed of the train is

Solution:

Speed of train = distance covered / time.

= (50 + 100) / 10

= 15 m/sec.

Example:How many seconds will a 500-meter-long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr?

Solution:

Relative speed:

= (63 - 3) km/hr

= 60 km/hr

= 60*5/18= 50/3 m/s

Time taken to pass the man= 500*3/50= 30 sec

Example:Two trains of equal lengths take 10 seconds and 15 seconds respectively to cross a telegraph post. If the length of each train be 120 m, in what time (in seconds) will they cross each other travelling in opposite direction?

Solution:

Speed of the first train= 120/10 = 12 m/s

Speed of the second train= 120/15 = 8 m/s

Relative speed= 12+8= 20m/s

Required time= (120+120)/20= 12 sec

Example:Two trains travel in opposite directions at 36 km/hr and 45 km/hr respectively. A man sitting in the slower train passes the faster train in 8 s. The length of the faster train is:

Solution:

As the trains travel in opposite direction then

Relative speed = 36+45- 81 km/hr= 81*5/18= 405/18 m/s

Hence, length of the faster train= 405*8/18= 180 m

Example:A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform?

Solution:

Speed of train= 54*5/18= 15 m/s

Length of the train = (15 x 20) m = 300 m.

Let the length of the platform be ‘x’ m.

Then, (300+x)/15 = 36

x= 240 m

Hence, the length of platform is 240 m

4.4 Boats and Streams- Concepts

There are a variety of sub concepts that are related to answering questions based on boat and streams concept. Given below are the four terms which are important for a candidate to know to understand the concept of streams.

- **Stream** – The moving water in a river is called a stream.
- **Upstream** – If the boat is flowing in the opposite direction to the stream, it is called upstream. In this case, the net speed of the boat is called the upstream speed
- **Downstream** – If the boat is flowing along the direction of the stream, it is called downstream. In this case, the net speed of the boat is called downstream speed
- **Still Water** – Under this circumstance the water is considered to be stationary, and the speed of the water is zero

This topic basically deals with calculating the speed of anything in the water when it flows along with the flow of water or in the opposite direction.

4.5 Downstream and Upstream-Formula

Given below are a few important formulas with the help of which you can solve the questions based on boat and streams.

Candidates must learn these formulas by heart to ensure they are able to answer the simple formula-based questions correctly and do not end up losing marks for direct questions.

- **Upstream(U) = (B−S) km/hr**, where “B” is the speed of the boat in still water and “S” is the speed of the stream
- **Downstream(D) = (B+S) Km/hr,** where “B” is the speed of the boat in still water and “S” is the speed of the stream
- **Speed of Boat in Still Water = ½ (Downstream Speed + Upstream Speed)**
- **Speed of Stream = ½ (Downstream Speed – Upstream Speed)**
- **Average Speed of Boat = {(Upstream Speed × Downstream Speed) / Boat’s Speed in Still Water}**

4.6 Types of Problems on Boats and Streams

The questions from this topic may usually be asked in four different formats. These include:

- **Time Based Questions** – The time taken by a boat to travel upstream or downstream may be asked with the speed of a boat in still water and speed of the stream given in the question

- **Speed Based Questions** – Questions to find the speed of the stream or the speed of the boat in still water may be asked
- **Questions on Average Speed** – With the speed of the boat upstream and downstream given in the question, the average speed of the boat may be asked
- **Questions Based on Distance** – The distance travelled by boat upstream or downstream may be asked

Example:A person can swim in water with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, what will be the time taken by the person to go 68 km downstream?

Solution:

Downstream Speed = (13+4) km/hr = 17 km/hr

To travel 68 km downstream.

Time taken = 68/17 = 4 hours

Example:In one hour, a boat goes 13 km/hr in the direction of the stream and 7 km/hr against the direction of the stream. What will be the speed of the boat in still water?

Solution:

According to the formula,

Speed of a boat in still water = ½ (DownstreamSpeed + UpstreamSpeed)

Speed of boat in still water = ½ (13+7) = ½ × 20 = 10 km/hr

Example:A boat sails 15 km of a river towards upstream in 5 hours. How long will it take to cover the same distance downstream, if the speed of current is one-fourth the speed of the boat in still water:

Solution:

Let the speed of boat in still tawer be B and speed of stream be S:

Upstream speed = B-S

Downstream speed = B+s

B-S = 15/5 = 3 km/h

Again B= 4S

Therefore B-S = 3= 3S

=> S = 1 and B= 4 km/h

Therefore B+S = 5km/h

Therefore, Time during downstream = 15/5 = 3h

Example:A man can row a certain distance against the stream in six hours. However, he would take two hours less to cover the same distance with the current. If the speed of the current is 2 Kmph, then what is the speed of the man in still water.

Let the speed of man in still tawer be B and speed of stream be S:

According to the question,

(B+S) *4= (B-S) *6

(B+2) *4= (B-2) *6

B= 10

Hence, the speed of man in still water is 10 km/h

Example:A man can row downstream at 12 Kmph and upstream at 8 Kmph. Find the ratio of the speed of the current to the speed of the man in still water?

Solution:

Speed of current= (12-8)/2= 2 kmph

Speed of man in still water = (12+8)/2= 10 kmph

Hence, ratio of the speed of the current to the speed of the man in still water= 2:10= 1:5

Example:A motorboat whose speed is 15 kmph in still water goes 30 km downstream and comes back in a total of 4hrs 30min. What is the speed of the stream?

Solution:

Let the speed of the stream be S km/hr

Upstream Speed = 15 -S

Downstream Speed = 15 +S

So, {30 / (15+S)} + {30 / (15-S)} = 4 ½ (4 hours 30 minutes)

$\Rightarrow$ {900 / $(225-S^2)$} = 9/2

$\Rightarrow 9S^2 = 225$

$\Rightarrow S^2 = 25$

$\Rightarrow S = 5$

Hence, the speed of the stream is 5kmph.

Example:A river flows at 4 km/hr. The speed of a boat in downstream is thrice the speed of that boat in upstream. Find out the speed of the boat in still water?

Solution:

Let the speed of upstream be 'x' kmph

So, the speed of downstream be '3x'

According to the question,

4= (3x-x)/2

So, x= 4

Hence, the speed of boat in still water= (3x+x)/2= 4x/2= 4*4/2= 8 kmph

4.7 <u>Races</u>

Races: A contest of speed in running, riding, driving, sailing, or rowing is called a race.

Racecourse: The ground or path on which contests are made is called a racecourse.

Starting Point: The point from which a race begins is known as a starting point.

Winning Point or Goal: The point set to bound a race is called a winning point or a goal.

Winner: The person who first reaches the winning point is called a winner.

Dead Heat Race: If all the persons contesting a race reach the goal exactly at the same time, the race is said to be dead heat race.

Start: Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 - 12) = 88 metres.

In a 100 race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats B by 12 m' means that while A runs 100 m, B runs (100 - 12) = 88 m.

Example:In a 100 m race, A can give B 10 m and C 28 m. In the same race B can give C:

Solution:

A: B = 100: 90.

A: C = 100: 72.

B:C= B/A * A/C = 90/100 * 100/72 = 90/72

When B runs 90 m, C runs 72 m.

When B runs 100 m, C runs (72*100/90) = 80 m

Hence,B can give C 20 m.

Example:In a 500 m race, the ratio of the speeds of two contestants A and B is 3: 4. A has a start of 140 m. Then, A wins by:

Solution:

To reach the winning post A will have to cover a distance of (500 - 140) m, i.e., 360 m.

While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers (4*360/3) =480 m

Thus, when A reaches the winning post, B covers 480 m and therefore remains 20 m behind.

Hence, A wins by 20 m.

Example:In 100 m race, A covers the distance in 36 seconds and B in 45 seconds. In this race A beats B by:

Solution:

Distance covered by B in 9 sec.= (100*9/45) = 20 m

Hence, A beats B by 20 m.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts Problem on Trains.
- We have learnt about Relative speed
- We have learnt to calculate crossing Time
- We have learnt about key concepts Boats and Streams
- We have learnt about races concepts

Keywords

- Relative speed
- Crossing Time
- Downstream
- Upstream
- Races

Self Assessment

1. A train 125 m long passes a man, running at 5 km/hr in the same direction in which the train is going, in 10 seconds. The speed of the train is?
   A. 45 kmph
   B. 50 kmph
   C. 54 kmph
   D. 55 kmph

2. A train crosses a platform of 150 m in 15 sec, same train crosses another platform of length 250 m in 20 sec. then find the length of the train?
   A. 150 m
   B. 160 m
   C. 165 m
   D. 155 m

3. Two trains, each 100 m long, moving in opposite directions, cross each other in 8 seconds. If one is moving twice as fast the other, then the speed of the faster train is:
   A. 30 km/hr.
   B. 45 km/hr.
   C. 60 km/hr.
   D. 75 km/hr.

4. Two trains travel in the same direction at 56 km and 29 km an hour and the faster train passes a man in the slower train in 16seconds. Find the length of the faster train
   A. 200 m
   B. 240 m
   C. 120 m
   D. 112 m

5. A train passes two persons walking in the same direction at a speed of 3 kmph and 5 kmph respectively in 10 seconds and 11 seconds respectively. The speed of the train is:
   A. 28 kmph
   B. 27 kmph
   C. 25 kmph
   D. 24 kmph

6. Two, trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is
   A. 2:3
   B. 4:3
   C. 6:7
   D. None

7. A boat running downstream covers 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water?
   A. 4 kmph
   B. 6 kmph
   C. 8 kmph
   D. Data inadequate

8. The current of a stream runs at 1 kmph. A motorboat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motorboat in the still water is?

A. 8 kmph
B. 6 kmph
C. 7.5 kmph
D. 5.5 kmph

9. A man takes twice as long to row a distance against the stream as to row the same distance along the stream. The ratio of the speed of the boat (in still water) and the stream is:
   A. 2:1
   B. 3:1
   C. 3:2
   D. 4:3

10. A boatman rows to a place at a distance 45 km and comes back in 20 hours. He finds that he can row 12 km with the stream in the same time as 4 km against the stream. Find the speed of the stream.
    A. 3 kmph
    B. 2.5 kmph
    C. 4 kmph
    D. 3.5 kmph

11. Find the speed of stream if a boat covers 36 km in downstream in 6 hours which is 3 hours less in covering the same distance in upstream?
    A. 1 kmph
    B. 1.5 kmph
    C. 0.75 kmph
    D. 0.5 kmph

12. In a 100 m race, A beats B by 10 m and C by 13 m. In a race of 180 m, B will beat C by:
    A. 5.4 m
    B. 4.5 m
    C. 5 m
    D. 6 m

13. In a 200 metres race A beats B by 35 m or 7 seconds. A's time over the course is:
    A. 40 sec
    B. 47 sec
    C. 33 sec
    D. None

14. A and B take part in 100 m race. A run at 5 kmph. A gives B a start of 8 m and still beats him by 8 seconds. The speed of B is:
    A. 5.15 kmph
    B. 4.14 kmph
    C. 4.25 kmph
    D. 4.4 kmph

15. In a stream running at 2 km/h, a motorboat goes 10 km upstream and returns to the starting point in 55 minutes. Find the speed (all in km/h) of the motorboat in still water.

A. 2
B. 11
C. 22
D. None

Answers for Self Assessment

1.	B	2.	A	3.	C	4.	C	5.	C
6.	B	7.	B	8.	B	9.	B	10.	A
11.	A	12.	D	13.	C	14.	B	15.	C

Review Questions

1. A train running at the speed of 60 km/hr. crosses a pole in 9 seconds. What is the length of the train?
2. Two trains of equal length are running on parallel lines in the same direction at 46 km/hr. and 36 km/hr. The faster train passes the slower train in 36 seconds. The length of each train is:
3. A man's speed with the current is 15 km/hr. and the speed of the current is 2.5 km/hr. The man's speed against the current is:
4. The ratio of the speed of the boat in still water to the speed of the current is 4:1. What is the ratio of the downstream speed of the boat to the upstream speed?
5. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m, C will beat B by:

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R S Aggarwal, S Chand Publishing
2. Magical Book on Quicker Math's by M Tyra, Banking Service Chronicle

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 05: Sequence and Series Completion

CONTENTS

Objectives

After studying this unit, you will be able to

- define the sequence and series.
- enlist different types of series.
- identify the pattern in the given series.
- differentiate between arithmetic and geometric progression.
- compute general term and sum of 'n' terms of arithmetic and geometric progression.
- analyze important points of arithmetic and geometric progression.
- solve different problems based on series completion, arithmetic progression, and geometric progression.

Introduction

In almost all competitive exams, problems related to sequence and series are asked. For example, problems based on finding missing number; wrong number; specific term of the series and sequence; complete and correct series; verification of sequence; computation of nth term, last term, sum of n terms, and nth term from end of arithmetic and geometric progression; consecutive terms of arithmetic and geometric progression;etc. This chapter includes information relating to different types of series, sequences, and progression. It will also help you in solving problems relating to series, sequences, and progression with easeand at fast rate.

5.1 Sequence

It is a set of numbers, follows a definite order/rule. Let us consider a sequence a_1, a_2, a_3, a_4, ……, a_n; where n is positive integer. For example 1, 3, 5, 7, ……, 21 or 2, 4, 6, 8, 10, ….., etc. A sequence may be finite or infinite. It is represented by $\{a_n\}$.

5.2 Series

A series is a sequence of numbers. These numbers are called terms of the sequence. All the terms of the sequence are arranged according to a certain predefined rule. After carefully studying the given series and finding the specific pattern in which the terms are changing, it is possible to find out the next term of the series.

5.3 Types of Series

Series may be classified as -

- Number Series
- Special Series
- Alpha–Numeric Series

Types of Number Series

Number series may be classified as -

- Arithmetic Series
- Arithmetic Series – Second Order
- Arithmetic Series – Third Order
- Geometric Series
- Arithmetico-Geometric Series
- Geometrico-Arithmetic Series
- Series of Squares, Cubes, …..

Arithmetic Series

An arithmetic series is one in which the difference between any two consecutive terms is always the same and is called the common difference. The successive number is obtained by using the formula - Successive Number = Previous Number (+) or (-) Fixed Number.

Example - 1, 4, 7, 10, 13, ...

- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = ... = 3$ (Common Difference)

Example - 13, 10, 7, 4, 1, ...

- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = ... = -3$ (Common Difference)

Arithmetic Series - Second Order

A series in which the difference between successive terms themselves form an arithmetic series is called an arithmetic series of second order.

Example - 1, 3, 7, 13, ...

- $T_2 - T_1 = 3 – 1 = 2$
- $T_3 - T_2 = 7 – 3 = 4$
- $T_4 - T_3 = 13 – 7 = 6$

Example - 2, 4, 6, ...

- $a_2 - a_1 = a_3 - a_2 = ... = 2$ (Common Difference)
- $T_5 = (13 + 8) = 21$

Arithmetic Series - Third Order

A series in which the difference between successive terms themselves form an arithmetic series of second order, is called an arithmetic series of third order.

Example - 2, 9, 17, 28, 44, ...

- T_2-T_1 = 9–2=7; T_3-T_2= 17–9=8; T_4-T_3 = 28–17=11; T_5-T_4 = 44–28=16

Example - 7, 8, 11, 16, ...

- a_2 - a_1 = 7 – 8 = 1; a_3 - a_2 = 11 - 8 = 3; a_4 - a_3 = 16 - 11 = 5
- 1, 3, 5, ……
- t_2 - t_1 = t_3 - t_2 = ... = 2 (Common Difference)
- T_6 = (44 + 23) = 67

Geometric Series

A geometric series is one in which the ratio of any two consecutive terms is always the same and is called the common ratio. Each successive number is by using the formula - Successive Number = Previous Number (x) or (÷) Fixed Number.

Example - 4, 8, 16, 32, ...

- $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$ = 2 (Common Ratio)

Example - 64, 32, 16, 8, ...

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{2}$(Common Ratio)

Arithmetic-Geometric Series

The given series is arithmetic-geometric series if each successive term is obtained by using the formula rule - Successive Term = (Previous term + Fixed Number) x (Another Fixed Number).

Example - 1, 9, 33, 105, ...

- T_2 - T_1 = 9 – 1 = 8; T_3 - T_2 = 33 – 9 = 24; T_4 - T_3 = 105 – 33 = 72
- 8, 24, 72, ... (Geometric Series)
- T_2 = 9 = (1 + 2) x 3; T_3 = 33 = (9 + 2) x 3; T_4 = 105 = (33 + 2) x 3
- T_5 = (105 + 2) × 3 = 107 x 3 = 321

Geometric-Arithmetic Series

The given series is geometric-arithmetic series if each successive term is obtained by using the formula rule - Successive Term = (Previous term (x)/(÷) Fixed Number) (+)/(-) (Another Fixed Number).

Example - 2, 5, 17, 65, ...

- T_2 - T_1 = 5 – 2 = 3; T_3 - T_2 = 17 – 5 = 12; T_4 - T_3 = 65 – 17 = 48
- 3, 12, 48, ... (Geometric Series, Common Ratio = 4)
- T_2 = 5 = (2 x 4) - 3; T_3 = 17 = (5 x 4) - 3; T_4 = 65 = (17 x 4) – 3
- T_5 = (65 x 4) - 3 = 260 – 3 = 257

Series of Squares, Cubes ……

Series of squares, cubes, etc. are simple powers of natural numbers i.e. squares, cubes, …. or their combinations.

Example - 4, 9, 16, 25, 36, 49, 64, ...

- $2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2$, ……..
- 2, 3, 4, 5, 6, 7, 8, …….
- Common Difference = 1
- $T_8 = (8+1)^2 = (9)^2 = 81$

Example - $\frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \frac{25}{216}$, ………

- $\frac{1^2}{2^3}, \frac{2^2}{3^3}, \frac{3^2}{4^3}, \frac{4^2}{5^3}, \frac{5^2}{6^3}$, ………
- $\frac{n^2}{(n+1)^3}$
- $T_6 = \frac{6^2}{7^3} = \frac{216}{343}$

Double Series

It consists of two series combined into a single series. The alternating terms of this series form an independent series

Example - 1, 2, 4, 6, 7, 18, 10, 54, ...

- 1, 4, 7, 10, ... - Arithmetic Series
- Common Difference = 4 – 1 = 7 – 4 = 10 – 7 = 3
- 2, 6, 18, 54, ... - Geometric Series
- Common Ratio = $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = 3$
- Next Term = 13 (i.e. 10 + 3)

Finding Wrong Term – Series

It is a series in which all others except one are similar in some respect. All terms follow the same pattern except one. That is wrong term.

Example - 5, 10, 17, 24, 37, 50, 65.

- $5 = 2^2 + 1; 10 = 3^2 + 1; 17 = 4^2 + 1; 26 = 5^2 + 1;\ 37 = 6^2 + 1; 50 = 7^2 + 1\ ; 65 = 8^2 + 1$

Finding Missing Term – Series

It is a series is given in which a blank space or question mark is provided in place of any one term of the series. The term at the blank space follows the same pattern as followed by other terms. We are required to find the missing term to replace the blank space or question mark

Example - 49, 56, 64, 72, ?, 90, 100

- $49 = 7^2;\ 56 = 7^2 + 7; 64 = 8^2; 72 = 8^2 + 8;\ ?\ ;\ 90 = 9^2 + 9; 10 = 10^2$
- $T_5 = 81 = 9^2$

Types of Special Series

The special series may be classified as -

- Series of Date/Time
- Numbers followed by their L.C.M. or H.C.F
- Numbers Followed by their Product
- By Use of Digit Sum

Series of Date/Time

Example-

- 3 - 9 - 2022, 13 - 9 - 2022, 23 - 9 - 2022, & 2 - 10 – 2022.
- Differs by 10 days.
- September, 2022 - 30 days
- 2 - 10 - 2004 replaced by 3 - 10 – 2022

Example –

- 1:00, 2:25, 3:40, 5:15, 6:40
- Difference - 1 hour 25 min.
- 3:40 should be replaced by 3:50.

Numbers Followed by Their L.C.M. or H.C.F

Example - 1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ?

- L.C.M. of 1, 2 & 3 = 6
- L.C.M. of 4, 5 & 6 = 60
- L.C.M. of 5, 6 & 7 = 210

Example - 8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ?

- H.C.F. of 8 and 4 = 4
- H.C.F. of 7 and 8 = 1
- H.C.F. of 3 and 9 = 3

H.C.F. of 2 and 1 = 1

Numbers Followed by Their Product

Example - 1, 3, 3, 9, 27, 243, ?

- 1 = 1
- 1 × 3 = 3
- 3 × 3 = 9
- 3 × 9 = 27
- 9 × 27 = 243
- Next Number = Product of Previous Two Numbers
- Next Number = 27 × 243 = 6561

Use of Digit Sum

Example - 11, 13, 17, 25, 32, ?

- 13 = 11 + (1 + 1)
- 17 = 13 + (1 + 3)
- 25 = 17 + (1 + 7)
- 32 = 25 + (2 + 5)
- Next Number =Previous Number + Sum of the digits of pervious number
- Next Number = 32 + (3 + 2) = 37

Alpha–Numeric Series

These series involve the use of both the letters of the alphabet as well as the numbers. It is a two-line series. One line is a number series while the other line is an alphabet series. The terms of both the series follow the same pattern/rule. One of these two series is completely known. We must find the required number of the incomplete series.Letters & Numbers

Example - 2, 7, 17, 37, 77,

3, a, b, c, d,

- 7 = 2 x 2 + 3; 17 = 7 x 2 + 3; 37 = 17 x 2 + 3; 77 = 37 x 2 + 3
- a = 3 x 2 + 3 = 9; b = 9 x 2 + 3 = 21; c = 21 x 2 + 3 = 45; d = 45 x 2 + 3 = 93

5.4 Problems Based on Sequence and Series

Example 1

Insert the missing number in the following series:

5, 8, 12, 17, 23,….., 38

Solution-

$T_2 - T_1 = 8 – 5 = 3$

$T_3 - T_2 = 12 – 8 = 4$

$T_4 - T_3 = 17 - 12 = 5$

$T_5 - T_4 = 23 - 17 = 6$

$T_6 - T_5 = X - 23 = 7$

$T_7 - T_6 = 38 - X = 8$

Example 2

Insert the missing number in the following series:

5, 8, 12, 17, 23,….., 38

Solution-

$T_2 - T_1 = 8 – 5 = 3$;$T_3 - T_2 = 12 – 8 = 4$; $T_4 - T_3 = 17 - 12 = 5$

$T_5 - T_4 = 23 - 17 = 6$; $T_6 - T_5 = X - 23 = 7$; $T_7 - T_6 = 38-X= 8$

30- 23 = 7 and 38 – 30 = 8

X or missing number = 30

5, 8, 12, 17, 23, 30, 38

Example 3

Insert the missing number in the given series:

4, 9, 20, 43, 90, _____

Solution-

$T_2 = 9 = 2 \times 4 + 1$

$T_3 = 20 = 2 \times 9 + 2$

$T_4 = 43 = 2 \times 20 + 3$

$T_5 = 90 = 2 \times 43 + 4$

T_6 = 2 × 90 + 5 = 185

4, 9, 20, 43, 90, 185

Example 4

Insert the missing number in the given series:

1, 1, 4, 8, 9, 27, 16, ___

Solution-

First Series – 1, 4, 9, 16, ……

or $1^2, 2^2, 3^2, 4^2$, …….

Second Series - 1, 8, 27,…..

or $1^3, 2^3, 3^3$, ...

or $1^3, 2^3, 3^3, 4^3$, ...

Missing Number = 64

Example 5

Fill in the missing number in the following series:

11, 10, ?, 100, 1001, 1000, 10001, ….

Solution-

T_2 = 10, T_4 = 100, T_6 = 1000, ……

T_1 = 11, T_3= ?,T_5 = 1001, T_7 = 10001, ……

T_3 = 101

11, 10, 101, 100, 1001, 1000, 10001, ….

Example 6

Find the fifth term in the following series:

99, 95, 86, 70,…….

Solution-

$T_2 - T_1$ = 95 – 99 = -4 = - 2^2

$T_3 - T_2$ = 86 - 95 = - 9 = - 3^2

$T_4 - T_3$ = 70 – 86 = - 16 = - 4^2

$T_5 - T_4$ = X – 70 = - 25 = - 5^2

T_5 = X = 70 – 25 = 45

99, 95, 86, 70, 45

Example 7

Find the number corresponding to question mark in the following series:

0, 3, 12, 30, ?, 105, 168

Solution-

$T_2 - T_1$ = 3 – 0 = 3

$T_3 - T_2$ = 12 - 3 = 9

$T_4 - T_3$ = 30 – 12 = 18

$T_5 - T_4$ = X – 30 = ?

$T_6 - T_5$= 105 – X = ?

$T_7 - T_6$= 168 - 105 = 63

0, 3, 12, 30, X, 105, 168

3 9 18 X – 30 105 – X 63

$a_2 - a_1$ = 9 – 3 = 6

$a_3 - a_2$ = 18 - 9 = 9

$a_4 - a_3$ = X – 30 – 18 = X – 48

$a_5 - a_4$ = 105 - X – (X - 30) ≈ 135 – 2X

$a_6 - a_5$ = 63 – (105 - X) = X - 42

$a_2 - a_1$ = 9 – 3 = 6;$a_3 - a_2$ = 18 - 9 = 9

$a_4 - a_3$ = X – 30 – 18 = X – 48;

$a_5 - a_4$ = 105 - X – (X – 30) = 135 – 2X

$a_6 - a_5$ = 63 – (105 – X) = X - 42

X = 60

3, 9, 60 – 48 = 12, 135 – 2 x 60 = 15, and 60 – 42 = 18

Example 8

Find out the wrong number in the following series:

455, 445, 465, 435, 485, 415, and 475

Solution-

455, 465, 485, and 475

465 - 455 = 10; 485 – 465 = 20; and 475 – 485 = -10

445, 435, and 415

435 – 445 = -10; and 415 - 435 = -20

Correct number = 485 + 30 = 515

Example 9

Find out the wrong number in the following series:

1, 5, 11, 19, 29, and 55

Solution-

5 - 1 = 4

11 –5 = 6

19 – 11 = 8

29 – 19 = 10

55 -29 = 26

Correct number = 29 + 12 = 41

Example 10

Find out the wrong number in the following series:

2, 4, 4, 16, 8, 256, and 64

Solution-

2, 4, 8, and 64

2^1, 2^2, 2^3, and 2^6

4, 16, and 256

2^2, 2^4, and 2^8

Correct number = 2^4 = 16

Example 11

In the following questions a number series is given. After the series a number is given followed by a, b, c, d and e. Complete the series starting with the number given following the sequence of the given series.

1, 9, 65, 393

2, a, b, c, d, e

Solution-

9 = 1 x 8 + 1

65 = 9 x 7 + 2

393 = 65 × 6 + 3

1, 9, 65, 393

2, a, b, c, d, e

9 = 1 x 8 + 1; 65 = 9 x 7 + 2; 393 = 65 × 6 + 3

a = 2 x 8 + 1 = 17

b = 17 × 7 + 2 = 121

c = 121 x 6 + 3 = 729

d = 729 x 5 + 4 = 3649

e = 3649 × 4 + 5 = 14601

Example 12

Find out the wrong number in the following series:

3, 10, 24, 54, 108, 220, and 444

Solution-

10 = 2 × 3 + 4

24 = 2 × 10 + 4

54 = 2 × 24 + 6

108 = 2 × 54 + 0

220 = 2 × 108 + 4

444 = 2 × 220 + 4

Correct number 52 = 2 × 24 + 4 and 108 = 2 x 52 + 4

5.5 Progression

It is a sequence of special type.

For examples,

- 3, 5, 7, 9, ..., 21
- 1, 4, 9, 16, ...
- 8, 5, 2, - 1, - 4, ...
- 3 + 5 + 7 + 9 + ... + 21
- 1 + 4 + 9 + 16 + ...
- 8 + 5 + 2 + (- 1) + ...

5.6 Arithmetic Progression (A.P.)

A sequence is said to be A.P. if terms of sequence increases/decreases by afixed number. This fixed number is called common difference.

Let First term = a

Common Difference = d

Then, A.P. is a, a + d, a + 2d, ..., a + (n - 1) d, ..

$T_n = a + (n - 1) d$

$d = T_n - T_{n-1}$

General Term of an A.P.

Let a and d be the first term and common difference.

A.P. is a, a + d, a + 2d, ..., a + (n - 1) d

$T_1 = a = a + (1 - 1) d$

$T_2 = a + d = a + (2 - 1) d$

$T_3 = a + 2d = a + (3 - 1) d$

………………………………………………………

$T_n = a + (n - 1) d$

Sum of 'n' Terms of an A.P.

Let a and d be the first term and common difference.

n = Terms

A.P. is a, a + d, a + 2d, ..., a + (n - 1) d

$S_n = a + (a + d) + (a + 2d) + ... + (a + (n - 1) d)$ ……. (1)

$S_n = (a + (n - 1) d) + …… (a + 2d) + (a + d) + a$ ……… (2)

$2S_n = n\{2a + (n - 1) d\}$

$S_n = \frac{n}{2} \{2a + (n - 1) d\}$

Did you know?

- If $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = …. = T_n - T_{n-1}$ = Constant

  Then $T_1, T_2, T_3, T_4, …., T_{n-1}$, and T_n are in A. P.
- Three numbers a, b, c are in A.P. iff b - a = c - b iff a + c = 2b
- Three terms of an A.P. a - d, a, and a + d
- Four terms of an A.P. are a - 3d, a - d, a + d, and a + 3d
- Five terms of an A.P. a - 2d, a - d, a, a + d, and a + 2d
- T_n or l = a + (n - 1) d

- If a= First Term, d=Common Difference, and m = Number of Terms of an A.P. then nth term from the end = (m - n + 1)th term from beginning i.e., T_{m-n+1} = a + (m – n +1 - 1) d or nth term from the end = T_{m-n+1} = a + (m - n) d
- $T_n = S_n - S_{n-1}$
- $S_n = \frac{n}{2}$ {2a + (n - 1) d}
  - $S_n = \frac{n}{2}$ {a + [a + (n - 1) d]}
  - $S_n = \frac{n}{2}$ {a + l} (Because T_n or l = a + (n - 1) d)
  - $S_n = \frac{n}{2}$ {a + l}
- $S_n = \frac{n}{2}$ {a + l}
- $S_n = \frac{n}{2}$ {a + (n - 1) d - (n - 1) d + l}
- $S_n = \frac{n}{2}$ {l - (n - 1) d + l} (Because l = a + (n - 1) d)
- $S_n = \frac{n}{2}$ [2l - (n - 1) d]

5.7 Geometeric Progression (G.P.)

A sequence is said to be G.P. if we get fixed ratio by dividing consecutive terms of the sequence except the first one. This fixed ratio is called common/constantratio.

Let First term = a

Common/Constant Ratio = r

Then, G.P. is a, ar, ar^2, ar^3, ..., ar^{n-1}, ..

$T_n = ar^{n-1}$

$r = \frac{T_n}{T_{n-1}}$

General Term – G. P.

Let a and r be the first term and common ratio.

G.P. is a, ar, ar^2, ar^3, ..., ar^{n-1}

T_1 = a = $ar^0 = ar^{1-1}$

$T_2 = ar^1 = ar^{2-1}$

$T_3 = ar^2 = ar^{3-1}$

……………………………………………

$T_n = ar^{n-1}$

Sum of ‘n’ Terms– G. P.

Let a and r be the first term and common ratio. n = Terms

G.P. is a, ar, ar^2, ar^3, ..., ar^{n-1}

$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$ ……. (1)

$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$ ……… (2)

$rS_n - S_n = ar^n - a$

$(r - 1) S_n = a(r^n - 1)$

$S_n = \frac{a(r^n - 1)}{(r - 1)}$

Did you know?

- If $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \ldots = \frac{T_n}{T_{n-1}}$ = Constant = r

  $T_1, T_2,\ T_3, T_4, \ldots, T_{n-1}$, and T_n - G. P.
- Three numbers a, b, c are in G.P.iff $\frac{b}{a} = \frac{c}{b}$ iff $b^2 = ac$
- Three terms in G.P. are $\frac{a}{r}$, a, and ar.
- Four terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}$, ar, and ar^3
- Five terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}$, a, ar, and ar^2
- T_n or $l = ar^{n-1}$
- If a =First Term, r = Common Ratio, and m= Number ofTerms of a G.P. then nth term from the end = (m - n + 1)th term from beginning or $T_{m-n+1} = ar^{m-n+1-1}$ or nth term from the end $= T_{m-n+1} = ar^{m-n}$
- nth term from the end in terms of last term 'l' & common ratio 'r' = $\frac{l}{r^{n-1}}$
- $S_n = \frac{a(r^n - 1)}{(r-1)}$ when r > 1
- $S_n = \frac{a(1 - r^n)}{(1-r)}$ when r < 1
- S_n = na when r = 1
- $S_n = \frac{lr - a}{r - a}$, where l = last term & r ≠ 1
- $S_\infty = \frac{a}{(1-r)}$ when | r | < 1 i.e., - 1 < r < 1

5.8 Problems Based on Progression

Example 13

Is the sequence given by T_n = 12n - 9 form an A. P.?

Solution-

T_n = 12n - 9

T_{n-1}= 12(n-1) - 9

T_{n-1} = 12n – 12 - 9

T_{n-1} = 12n - 21

$T_n - T_{n-1}$ = (12n - 9) – (12n – 21)

= 12n - 9 – 12n + 21

= 12

Example 14

Find the 15th term of the sequence 12, 7, 2, - 3, - 8, ...

Solution-

First term = a = 12

Common Difference = d = - 5

n = 15

T_n = a + (n - 1) d

T_{15} = 12 + (15 - 1) (-5)

T_{15} = 12 – 70

T_{15} = - 58

Example 15

Find the sum of the series 5 + 9 + 13 + ... to 50 terms.

Solution-

a = 5, d = 9 - 5 = 4, and n = 50

$S_n = \frac{n}{2}$ {2a + (n - 1) d}

$S_{50} = \frac{50}{2}$ {2 x 5 + (50 - 1) x 4}

S_{50} = 25{10 + (49) x 4}

S_{50} = 25{10 + 196}

S_{50} = 25{206}

S_{50} = 5150

Example 16

Find the sum of 30 terms of an A.P., whose first and last terms are 25 and 216 respectively.

Solution-

a = 25, l = 216, and n = 30

$S_n = \frac{n}{2}$ {a + l}

$S_{30} = \frac{30}{2}$ {25 + 216}

S_{30} = 15{241}

S_{30} = 3615

Example 17

Find the sum of 20 terms of an A.P., whose common difference and last term are 7 and 93 respectively.

Solution-

d = 7, l = 93, and n = 20

$S_n = \frac{n}{2}$ [2l - (n - 1) d]

$S_{20} = \frac{20}{2}$ [2 x 93 - (20 - 1) x 7]

S_{20} = 10 [186 - 133]

S_{20} = 10 [53]

S_{20} = 530

Example 18

If sum of n terms of an A. P. is given by S_n = 100 n^2 + 25, then find its nth term.

Solution-

S_n = 100 n^2 + 25

$S_{n-1} = 100\ (n-1)^2 + 25$

$$T_n = S_n - S_{n-1}$$

$$T_n = 100\ n^2 + 25 - [100\ (n-1)^2 + 25]$$

$$T_n = 100\ n^2 + 25 - [100(n^2 - 2n + 1) + 25]$$

$$T_n = 100\ n^2 + 25 - [100n^2 - 200n + 100\ + 25]$$

$$T_n = 100\ n^2 + 25 - 100n^2 + 200n - 100\ - 25$$

$$T_n = 200n - 100$$

Example 19

Determine x so that x + 2, 4x - 6 and 3x - 2 are the three consecutive terms of an A.P.

Solution-

Three numbers a, b, c are in A.P. iff b - a = c - b iff a + c = 2b

x + 2, 4x – 6, and 3x – 2

a b c

$x + 2 + 3x - 2 = 2(4x - 6)$

$4x = 8x - 12$

$12 = 8x - 4x$

$4x = 12$

$x = 3$

Three consecutive terms are x + 2, 4x - 6 and 3x – 2

i.e., 3 + 2, 4(3) - 6 and 3(3) – 2

i.e., 5, 6 and 7

Example 20

Find the 7th term from the end of the A. P. of 50 terms whose first term and common difference are 5 and 8 respectively.

Solution-

n^{th} term from the end = $(m - n + 1)^{th}$ term from beginning

$T_{m-n+1} = a + (m - n)\ d$

a = 5, d = 8, m = 50, and n = 7

$T_{50-7+1} = 5 + (50 - 7) \times 8$

$T_{44} = 5 + (43) \times 8$

$T_{44} = 5 + 344$

$T_{44} = 349$

Example 21

Is the sequence 3, -6, 12, -24, ……. form a G. P.?

Solution-

$T_1 = 3, T_2 = -6, T_3 = 12, T_4 = -24, T_5 = 48, \ldots.$

$\frac{T_2}{T_1} = \frac{-6}{3} = -2$

$\frac{T_3}{T_2} = \frac{12}{-6} = -2$

$\frac{T_4}{T_3} = \frac{-24}{12} = -2$

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -2$ (G.P.)

Example 22

Find the 9th term of the sequence 2, 4, 8, 16, 32 ....

Solution-

First term = a = 2

Common ratio = r = $\frac{4}{2}$ = 2

n = 9

$T_n = ar^{n-1}$

$T_9 = (2)\, 2^{9-1}$

$T_9 = (2)\, 2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$T_9 = 512$

Example 23

Find the sum of 7 terms of the series $\frac{1}{4} + \frac{1}{2} + 1$ ……...

Solution-

$a = \frac{1}{4}$, $r = \frac{\frac{1}{2}}{\frac{1}{4}}$ or $\frac{1}{\frac{1}{2}}$ = 2 (> 1), and n = 7

$S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{\frac{1}{4}(2^7 - 1)}{(2 - 1)}$

$S_n = \frac{\frac{1}{4}(128 - 1)}{1}$

$S_n = \frac{127}{4}$

Example 24

Find the sum of 12 terms of a G.P., whose first term, common ratio, and last terms are 3, (-2) and 48 respectively.

Solution-

a = 3, r = -2, l = 48, and n = 12

$S_n = \frac{lr - a}{r - a}$

$S_n = \frac{48 \times (-2) - 3}{-2 - 3}$

$S_n = \frac{(-96) - 3}{-5} = \frac{-99}{-5}$

$S_n = \frac{99}{5}$

Example 25

Find the sum of 5 terms of a G.P., whose first term, and common ratio are 3 and (-2) respectively.

Solution-a = 3, r = -2 (< 1), and n = 5

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$S_5 = \frac{3(1 - (-2)^5)}{(1 -(-2))}$$

$$S_5 = \frac{3(1 -(-32))}{(1 -(-2))} = \frac{3(1 + 32)}{(1 + 2)} = \frac{3(33)}{(3)}$$

$$S_5 = 33$$

Example 26

Find the sum of the infinite sequence 9, - 3, 1, $\frac{-1}{3}$, …..

Solution-

a = 9, r = $\frac{-3}{9} = \frac{-1}{3}$ where $\left|\frac{-1}{3}\right| < 1$ i.e., $\frac{-1}{3} < r < \frac{1}{3}$

$$S_\infty = \frac{a}{(1 - r)}$$

$$S_\infty = \frac{9}{(1 -(\frac{-1}{3}))}$$

$$S_\infty = \frac{9}{(1 + \frac{1}{3})} = \frac{9}{(\frac{3+1}{3})} = \frac{9 \times 3}{4}$$

$$S_\infty = \frac{27}{4}$$

Example 27

Determine x so that x, x + 2, and x + 6 are the three consecutive terms of a G.P.

Solution-

Three numbers a, b, c are in G.P. iff $\frac{b}{a} = \frac{c}{b}$ iff $b^2 = ac$

x,	x + 2,	and	x + 6
a	b		c

$$(x + 2)^2 = x (x + 6)$$

$$x^2 + 4x + 4 = x^2 + 6x$$

$$4 = 6x – 4x$$

$$2x = 4$$

$$x = 2$$

Three consecutive terms are x, x + 2, and x + 6

i.e., 2, 2 + 2, and 2 + 6

i.e., 2, 4, and 8

Example 28

Find the 5th term from the end of the G. P. of 15 terms whose first term and common ratio are 9 and 3 respectively.

Solution-

n^{th} term from the end = $(m - n + 1)^{th}$ term from beginning

nth term from the end = $T_{m-n+1} = ar^{m-n}$ {Because $T_n = ar^{n-1}$}

$T_{m-n+1} = ar^{m-n}$

a = 9, r = 3, m = 15, and n = 5

$T_{15-5+1} = 9 \times (3)^{15-5}$

$T_{11} = 9 \times (3)^{10}$

$T_{11} = 9 \times 59049$ or $(3)^2 \times (3)^{10}$

$T_{11} = 531441$ or $(3)^{12}$

Example 29

Find the 10th term from the end of the G. P. with last term $\frac{512}{729}$ and common ratio $\frac{2}{3}$.

Solution-

nth term from the end in terms of last term 'l' & common ratio 'r' is $T_n = \frac{l}{r^{n-1}}$

$l = \frac{512}{729}$, $r = \frac{2}{3}$, and n = 10

$T_{10} = \frac{\frac{512}{729}}{\left(\frac{2}{3}\right)^{10-1}}$

$l = \frac{512}{729}$, $r = \frac{2}{3}$, and n = 10

$T_{10} = \frac{\frac{512}{729}}{\left(\frac{2}{3}\right)^{10-1}} = \frac{\frac{(2)^9}{(3)^6}}{\frac{(2)^9}{(3)^9}}$

$T_{10} = \frac{(2)^{9-9}}{(3)^{6-9}} = \frac{(2)^0}{(3)^{-3}}$

$T_{10} = 1 \times (3)^3$

$T_{10} = 27$

Summary

In short,

- If $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \ldots = T_n - T_{n-1}$ = Constant

  Then $T_1, T_2, T_3, T_4, \ldots, T_{n-1}$, and T_n are in A. P.
- Three numbers a, b, c are in A.P. iff b - a = c - b iff a + c = 2b
- Three terms of an A.P. a - d, a, and a + d
- Four terms of an A.P. are a - 3d, a - d, a + d, and a + 3d
- Five terms of an A.P. a - 2d, a - d, a, a + d, and a + 2d
- T_n or l = a + (n - 1) d
- If a= First Term, d=Common Difference, and m = Number of Terms of an A.P. then nth term from the end = (m - n + 1)th term from beginning i.e., T_{m-n+1} = a + (m – n +1 - 1) d or nth term from the end $= T_{m-n+1}$ = a + (m - n) d
- $T_n = S_n - S_{n-1}$
- $S_n = \frac{n}{2}\{2a + (n-1)d\}$
  - $S_n = \frac{n}{2}\{a + [a + (n-1)d]\}$
  - $S_n = \frac{n}{2}\{a + l\}$ (Because T_n or l = a + (n - 1) d)

- $S_n = \frac{n}{2}\{a + l\}$

- $S_n = \frac{n}{2}\{a + l\}$
- $S_n = \frac{n}{2}\{a + (n - 1) d - (n - 1) d + l\}$
- $S_n = \frac{n}{2}\{l - (n - 1) d + l\}$ (Because $l = a + (n - 1) d$)
- $S_n = \frac{n}{2}[2l - (n - 1) d]$
- If $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \ldots = \frac{T_n}{T_{n-1}}$ = Constant = r

  $T_1, T_2, T_3, T_4, \ldots, T_{n-1}$, and T_n - G. P.
- Three numbers a, b, c are in G.P.iff $\frac{b}{a} = \frac{c}{b}$ iff $b^2 = ac$
- Three terms in G.P. are $\frac{a}{r}$, a, and ar.
- Four terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}$, ar, and ar^3
- Five terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}$, a, ar, and ar^2
- T_n or $l = ar^{n-1}$
- If a =First Term, r = Common Ratio, and m= Number of Terms of a G.P. then nth term from the end = (m - n + 1)th term from beginning or $T_{m-n+1} = ar^{m-n+1-1}$ or nth term from the end $= T_{m-n+1} = ar^{m-n}$
- nth term from the end in terms of last term 'l' & common ratio 'r' = $\frac{l}{r^{n-1}}$
- $S_n = \frac{a(r^n - 1)}{(r - 1)}$ when r > 1
- $S_n = \frac{a(1 - r^n)}{(1 - r)}$ when r < 1
- S_n = na when r = 1
- $S_n = \frac{lr - a}{r - a}$, where l = last term & $r \neq 1$
- $S_\infty = \frac{a}{(1 - r)}$ when $|r| < 1$ i.e., $-1 < r < 1$

Keywords

Sequence is a set of numbers, follows a definite order/rule.

Series is a sequence of numbers.

Arithmetic series is one in which the difference between any two consecutive terms is always the same.

Geometric series is one in which the ratio of any two consecutive terms is always the same.

Progression is a sequence of special type.

A sequence is said to be **A.P.** if terms of sequence increases/decreases by a fixed number.

A sequence is said to be **G.P.** if we get fixed ratio by dividing consecutive terms of the sequence except the first one.

Self Assessment

1. Insert the missing number 5, 8, 12, 17, 23, , 38

A. 29

B. 30

C. 32

D. 25

2. Fill in the missing number 5, 18, 10, 12, 15, ?

A. 4

B. 8

C. 6

D. 1

3. Fill in the missing number 2, 6, 3, 4, 20, 5, 6, ?, 7

A. 25

B. 42

C. 24

D. 18

4. In the following number series a wrong number is given. Find out the wrong number.

455, 445, 465, 435, 485, 415, 475

A. 475

B. 465

C. 435

D. 455

5. Which of the following numbers will come in place of (C)?

1 9 65 393

2 (A) (B) (C) (D) (E)

A. 490

B. 729

C. 854

D. 734

6. What is the sum of the first 17 terms of an arithmetic progression if the first term is 20 and last

term is 28?

A. 68

B. 156

C. 142

D. 242

7. What is the sum of the first 9 terms of an arithmetic progression if the first terms is 7 and the last term is 55?

A. 219

B. 137

C. 231

D. 279

8. The 3rd and 7th term of an arithmetic progression are –9 and 11 respectively. What is the 15th term?

A. 28
B. 87
C. 51
D. 17

9. Find the Wrong number in the following number series.

3, 7, 16, 35, 70, 153

A. 70
B. 16
C. 153
D. 35

10. What will come in place of question mark (?) in the given number series?

123 140 106 157 89 ?

A. 174
B. 139
C. 198
D. 169

11. Find out the wrong number in the sequence: 40960, 10240, 2560, 640, 200, 40, 10

A. 2560
B. 200
C. 640
D. 40

12. Find the value of x in the series 2, 6, 30, 210, x, 30030, ...

A. 2310
B. 1890
C. 2520
D. 2730

13. Insert the missing number 3, 18, 12, 72, 66, 396, ?

A. 300
B. 380
C. 350
D. 390

14. In the following number series only one number is wrong. Find out the wrong number.

7, 12, 40, 222, 1742, 17390, 208608

A. 7
B. 12
C. 40
D. 1742

15. In the following number series only one number is wrong. Find out the wrong number.

6, 91, 584, 2935, 11756, 35277, 70558

A. 91
B. 70558
C. 584
D. 2935

Answers for Self Assessment

1.	B	2.	C	3.	B	4.	A	5.	B
6.	A	7.	D	8.	C	9.	A	10.	A
11.	B	12.	A	13.	D	14.	D	15.	C

Review Questions

In each question below (1-3), a number series is given in which one number is wrong. Find out the wrong number

1. 484, 240, 120, 57, 26.5, 11.25, 3.625
2. 5, 7, 16, 57, 244, 1245, 7506
3. 6, 7, 16, 41, 90, 154, 292
4. Find the 15^{th} term of the sequence 12, 7, 2, - 3, - 8, ...
5. Find the sum of the series 15 + 19 + 23 + ... to 50 terms.
6. Find the sum of 50 terms of an A.P., whose first and last terms are 25 and 216 respectively.
7. Determine x so that x + 4, 8x - 10 and 6x - 4 are the three consecutive terms of an A.P.
8. Find the sum of 20 terms of the series $\frac{1}{4} + \frac{1}{2} + 1$ ……...
9. Find the sum of 15 terms of a G.P., whose first term, and common ratio are 3 and (-2) respectively.
10. Find the 15^{th} term from the end of the G. P. with last term $\frac{512}{729}$ and common ratio $\frac{2}{3}$.

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
2. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal. S. Chand & Co Ltd. (2010).
3. Quantitative Aptitude for Competitive Examinations by Dinesh Khattar, Pearson Education (2020).

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Unit 06: Alphabet Test and Logical sequence of word

CONTENTS

Objectives

After studying this unit, you will be able tosolve different questions of alphabet test and logical sequence.

How to arrange words according to dictionary and their meaning.

Introduction

Arranging words in alphabetical order implies 'to arrange them in the order as they appear in a dictionary', i.e., as per the order in which the beginning letters of these words appear in the English alphabet.

6.1 Alphabet Test

Alphabet Test is one of the easiest and important concepts of General Mental Ability Segment of Reasoning. In this type of question, students are asked to find the place of an alphabet or a word based on the different types of arrangement. Here, you will learn different forms of Alphabet Arrangements like arranging Words in Alphabetical Order, problems based on Letter-Word, Alphabetical Quibble and New Word Formation, etc.

6.2 Type of Alphabet Arrangement

TYPE 1: Arranging Words in ALPHABETICAL ORDER

Arranging words in alphabetical order implies arranging words as per the order in which the beginning letters of those words appear in the English Alphabet or to arrange them in the order as they appear in a dictionary.

Example: In the following question five words are given. Find out which word will come in the middle, if all the five words are arranged alphabetically as in a dictionary.

a) Savour

b) Save

c) Savage

d) Sausage

e) Saviour

Answer: b)

Explanation: If we arrange the words in alphabetical order, then the word 'Save' will come in the middle:

Sausage, Savage, Save, Saviour, Savour

TYPE 2: Problems based on LETTER-WORD

Example: How many such letters are there in the word "ACCELERATION", each of which is as far away from the beginning of the word as it is from the beginning of the English Alphabet?

a) None

b) One

c) Two

d) Three

Answer: b)

Explanation: Clearly, C is the third letter in the word "ACCELERATION" as well as in the English Alphabet. Therefore, there is only one such letter.

TYPE 3: Problems based on ALPHABETICAL QUIBBLE

Example: Answer this question based on the following English Alphabet:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

If in the English alphabet every fourth letter is replaced by the symbol (#), which of the following would be ninth to the left of the fourteenth element from the left?

a) E

b) #

c) W

d) F

Answer: a)

Explanation: If every fourth alphabet is replaced by # then new series becomes:

A B C # E F G # I J K # M N O # Q R S # U V W # Y Z

Here, the fourteenth element from the left is N. The ninth element to the left of N is E.

TYPE 4: NEW WORD FORMATION using letters of a word given in the problem

Example: If it is possible to make meaningful words with the second, the third, the sixth and the eighth letter of the word 'FRAGMENT', using each letter only once, then how many new words can be formed with these letters?

a) 1

b) 2

c) 3

d) 4

Answer: c)

Explanation: The second, the third, the sixth and the eighth letter of the word FRAGMENT are R, A, E and T, respectively. The three new words formed will be TEAR, TARE and RATE.

6.3 Logical Sequence of words

'Logical Order of Words' is basically the arrangement of words according to a certain order which can be their size, occurrence, dictionary order etc.

In these types of questions, four/five/six words are given and the candidate is required to arrange the given words either in a logical sequence or in an order according to dictionary.

6.4 Type of Logical Sequence Questions

Type 1: Arrange According to Logic

As the name implies, in this type of questions, a sequence is formed with a certain number of words given in such a way that it gives a logical step-by-step completion of the process or the activity described.

Example: Arrange the given words in a logical and meaningful order.

1. Frog 2. Eagle 3. Grasshopper 4. Snake 5. Grass

(a) 3, 4, 2, 5, 1 (b) 1, 3, 5, 2, 4

(c) 5, 3, 1, 4, 2 (d) 5, 3, 4, 2, 1

Solution: (c) From the given words, it is deduced that grass is eaten by grasshopper, grasshopper is eaten by frog, frog is eaten by snake and finally, eagle eats snake. So, the correct logical arrangement of words is 5, 3, 1, 4, 2.

Type 2: Arrangement According to Dictionary

Arranging words in alphabetical order implies 'to arrange them in the order as they appear in a dictionary'. For this arrangement, first we shall take the first letter of each word and then arrange the words in the order in which they appear in the English alphabet, then take the second letter and so on.

Example: Arrange the following words according to dictionary.

1. Fenestration 2. Feather 3. Feed head 4. Feature 5. Feminine

(a) 4, 2, 3, 5, 1

(b) 2, 4, 1, 5, 3

(c) 2, 4, 3, 5, 1

(d) 4, 2, 3, 1, 5

Solution: (c) Sequence of given words as per dictionary is a follows

Feather --->Feature ----> Feed head -----> Feminine ----> Fenestration, i.e. 2, 4, 3, 5, 1

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts of Alphabet test and logical sequence.
- We have learnt tricks to solve different types of Alphabet test and logical sequence.
- We have learnt to solve the questions with meaning and according to dictionary.

Keywords

- Alphabet test
- Logical Sequence

SelfAssessment

Directions : Which one of the given responses would be a meaningful order of the following words?

1. (a) Honey(b) Flower(c) Bee(d) Wax.

A. b, a, d, c
B. b, c, a, d
C. d, c, b, a
D. a, c, d, b

2. (a) Plant(b) Food(c) Seed(d) Leaf(e) Flower

A. a, c, d, e, b
B. c, b, d, e, a
C. c, a, d, e, b
D. e, d, c, b, a

3. a. Probation, b. Interview,c. Selection, d. Appointment,e. Advertisement,f. Application

A. e, f, c, b, d, a
B. e, f, d, b, c, a
C. f, e, d, b, c, a
D. e, f, b, c, d, a

4. a. Doctorb. Feverc. Prescribed. Diagnosee. Medicine

A. b, a, c, d, e
B. a, d, c, b, e
C. b, a, d, c, e
D. b, d, c, e, a

5. Which one of the given responses would be a meaningful orderof the following ?

1. Orange2. Indigo3. Red4. Blue5. Green6. Yellow7. Violet

A. 7, 2, 4, 5, 6, 1, 3
B. 7, 2, 4, 6, 5, 1, 3
C. 7, 2, 6, 4, 5, 1, 3
D. 7, 2, 6, 4, 1, 5, 3

6. Arrange the following words in ameaningful order :

1. Brother
2. Husband
3. Father
4. Son
5. Son-in-law

A. 3, 2, 1, 5, 4
B. 4, 1, 2, 5, 3
C. 4, 1, 5, 2, 3
D. 3, 1, 4, 2, 5

7. Which one of the following wordswill come fourth in the Dictionary?

Propriety, Proposition, Prosecute,Proposal, Prosody.

A. Proposition
B. Prosody
C. Proposal
D. Prosecute

8. Arrange the following words according to English Dictionary.

(1) PREMONITION
(2) PRELUDE
(3) PREMICE
(4) PRELIMINARY
(5) PREMIUM

A. 42153
B. 24351
C. 42351
D. 24135

9. Arrange the following words according to dictionary :

a.Brushb. Breadc.Broadd. Bordere.Butter

A. d, a, b, c, e
B. d, b, c, a, e
C. d, b, a, c, e
D. d, c, b, a, e

10. Arrange the following words asper order in the dictionary :

a. Detach

b. Devise

c. Denote

d. Digest

e. Depict

A. e,d,c,b,a
B. c,e,a,b,d
C. c,b,a,e,d
D. e,b,d,a,c

11. Arrange the following words asper order in the dictionary.

1. Billian

2. Bifurcate

3. Bilateral

4. Bilirubin

A. 2, 1, 3, 4
B. 4, 3, 2, 1
C. 2, 3, 4, 1
D. 2, 3, 1, 4

12. Arrange the following accordingto dictionary.

1.Fenestration

2.Feather

3. Feed head

4.Feature

5. Feminine

A. 4, 2, 3, 5, 1
B. 4, 2, 3, 1, 5
C. 2, 4, 3, 5, 1
D. 2, 4, 1, 5, 3

13. Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

E H R A S P

1 2 3 4 5 6

A. 2, 4, 6, 1, 3, 5

B. 3, 4, 2, 1, 6, 5

C. 5, 2, 4, 6, 1, 3

D. 6, 2, 3, 4, 5, 1

14. Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

T L P N A E

1 2 3 4 5 6

A. 3,2,5,4,1,6

B. 3,2,5,4,6,1

C. 4,5,3,6,2,1

D. 4,6,1,3,5,2

15. R E 5 D A P $ 3 T I Q 7 9 B # 2 K % U 1 M W 4 * J 8 N

Which of the following is exactly in the middle between 3 and 1 in the above arrangement?

A. B

B. K

C. 9

D. #

Answers forSelfAssessment

l.	B	2.	C	3.	D	4.	C	5.	A
6.	A	7.	D	8.	C	9.	B	10.	B
11.	C	12.	C	13.	D	14.	B	15.	A

Review Questions

1.Arrange the following words asper their order in the dictionary :

1. PHYSICAL

2. PHYSICS

3. PHYSIOLOGY

4. PHYSICIAN

5. PHONE

(1) 5, 1, 4, 2, 3

(2) 5, 1, 2, 3, 4

(3) 1, 4, 5, 2, 3

(4) 1, 5, 4, 3, 2

2.Arrange the following words asper order in the dictionary :

1. Ambitions

2. Ambiguous

3. Ambiguity

4. Animation

5. Animal

(1) 3 ,2 , 4 , 1 , 5

(2) 3 , 2 , 5 , 4 , 1

(3) 3 , 2 , 1 , 5 , 4

(4) 3 , 2 , 4 , 5 , 1

3.Arrange the following words asper order in the dictionary :

1. Brittle
2. Brisk
3. Bright
4. Bride

(1) 4, 3, 2, 1

(2) 1, 2, 3, 4

(3) 2, 3, 4, 1

(4) 4, 2, 1, 3

4.Arrange the following words asper order in the dictionary :

1. Preach
2. Praise
3. Precinet
4. Precept
5. Precede

(1) 2, 1, 5, 4, 3

(2) 2, 1, 3, 4, 5

(3) 2, 5, 1, 4, 3

(4) 1, 2, 5, 4, 3

5.Arrange the following words asper order in the dictionary :

1. Follicle
2. Folk
3. Follow
4. Foliage

(1) 4, 2, 1, 3

(2) 3, 4, 2, 1

(3) 4, 3, 1, 2

(4) 2, 4, 3, 1

6.Arrange the following words asper order in the dictionary

1. Maternity
2. Matriarchy
3. Matchbox
4. Matricide

(1) 3, 1, 2, 4

(2) 4, 3, 1, 2

(3) 3, 4, 1, 2

(4) 1, 3, 4, 2

7.Arrange the following words asper order in the dictionary

1. Launderette

2. Laughter

3. Laundry

4. Launch

(1) 4, 1, 2, 3

(2) 1, 3, 2, 4

(3) 4, 2, 1, 3

(4) 2, 4, 1, 3

8.Arrange the following words asper order in the dictionary.

1.Forecast

2. Forget

3.Foreign

4. Forsook

5.Force

(1) 3, 5, 1, 2, 4 (2) 5, 1, 3, 2, 4

(3) 5, 1, 3, 4, 2 (4) 5, 1, 2, 3, 4

9.Arrange the following words asper order in the dictionary.

1. Continuation

2. Contention

3. Contain

4. Continuous

5. Count

(1) 32415

(2) 32451

(3) 31245

(4) 32145

10.Which one of the given responses would be a meaningful orderof the following?

1. Stone

2. Sand

3. Rock

4. Boulder

5. Hill

(1) 2, 1, 3, 4, 5 (2) 5, 3, 2, 1, 4

(3) 5, 4, 2, 1, 3 (4) 1, 4, 2, 3, 5

11.Arrange the words given below

in a meaningful sequence :

1. Printer

2. Publisher

3. Writer

4. Editor

5. Seller

(1) 3, 4, 2, 1, 5 (2) 3, 4, 1, 2, 5

(3) 2, 4, 3, 5, 1 (4) 2, 3, 4, 1, 5

12. .Arrange the words given belowin a meaningful sequence :

1. Crop 2. Root 3. Stem

4. Seed 5. Flower

(1) 2 3 5 1 4

(2) 2 4 5 1 3

(3) 2 3 4 1 5

(4) 4 2 3 5 1

13.Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

N A E H L D

1 2 3 4 5 6

a. 2, 1, 6, 4, 3, 5
b. 2, 6, 4, 3, 5, 1
c. 4, 2, 1, 6, 5, 3
d. 4, 3, 6, 5, 2, 1

14.How many meaningful words can be formed using the first, the third, the fifth and the sixth letters of the word TRADEMARK using each letter only once in each word?

a. One
b. Two
c. Three
d. Four
e. More than four

15. £= β F 2 * K S 7 5 # $ P L V 8 @ M U E 6 ∞Q G © 9 3 & T Y ¥

How many such letters are there in the arrangement each of which is either immediately preceded by a symbol or immediately followed by a number, but not both?

a. Three

b. Four

c. Five

d. Six

e. None of these

Further Reading

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing

2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Web Links

1. https://www.examveda.com/mcq-question-on-competitive-reasoning/
2. https://www.hitbullseye.com/Reasoning

Unit 07: Coding-Decoding

CONTENTS

Objectives

After studying this unit, you will be able to

- Understand different types of Coding-Decoding
- Understand how to solve different coding-decoding based problems
- Understand Logic to solve Problems.

Introduction

Coding-Decoding is a process of encrypt or decrypt any word, letter, or sentence in a pattern or code based on some sets of rules. Coding and Decoding of an information done with some rules or patterns. Coding-Decoding helps candidates to improve their logical reasoning skills and ability to focus.

Coding is a process used to encrypt a word, a number in a particular code or pattern based on some set of rules.

Decoding is a process to decrypt the pattern into its original form from the given codes.

Coding-Decoding Questions are used to judge the ability to decipher the rule or pattern, which is applied at the time of coding the information.

To solve the Questions based on Coding-Decoding, first need to remember the positions of all Alphabetical letters, both in forward and backward order, which is given below.

Positional Value of Alphabets in Forward order

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

Positional Value of Alphabets in Backward order

A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	12	11	10	9	8	7	6	5	4	3	2	1

The pattern below is very useful to find many types of question in Alphabet Series when we write the last 13 alphabets in front of the first 13 alphabets of the English:

A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

Preparation Tips for Coding-Decoding Section

Few tips are given below which will help candidates solve the coding decoding questions

- Read the question carefully and write down key codes
- Try to decode the coding given in the question. Candidates must note that similar type of decoding can be applied to various questions
- Try to simplify the code by using the elimination method and remove the unwanted values
- Solve more and more practice questions to understand the concept even better and to score more.

7.1 Letter Coding

Letter coding-decoding in which the letter of words is replaced by certain other letters according to specific patterns/rules to form a code.

In this type of coding-decoding the alphabets of word coded with the help of different operations such as addition, subtraction, interchanging and so on. Candidates need to find the code of another word using the same operations.

Example:If EARTH is coded as FBSUI,how is MOTHERcoded as?

Logic: In this, each letter is moved one letters forward.So, the code for the given word MOTHER is NPUIFS.

Example:If COURSE is coded as GSYVWI, how is NATIONcoded as?

Logic: In this, each letter is moved four letters forward.So, the code for the given word NATION is REXMSR.

Example:In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?

Logic: The letter of the word is written in a reverse order and then eachletter is moved one step backward to obtain the code. So, the code for the given word TIGER is QDFHS.

Example:In a certain code, PARTNER is written as NCPVLGP. Inthe same code, what will NATION be written as?

Logic: The letters at odd positions are each moved two steps backward and those at even positionsare each moved two steps forward to obtain thecorresponding letters of the code.

So, the code for the given word NATION is LCRKMP.

Example:If EARTH is written as FCUXM in a certain code. How is TEMPLE written in that code?

Logic: the letters of given word are respectively moved one, two, three, four, and so on steps forward to obtain the corresponding letters of the code.

So, the code for the given word TEMPLE is UGPTQK.

Example:If BOMBAY is written as MYMYMY, how will TAMIL NADU be written in that code?

Logic: The letters at the third and sixth places are repeated thrice to codeBOMBAY as MYMYMY. Similarly, the letters at the third, sixth and ninthplaces are repeated thrice to code TAMIL NADU as MNUMNUMNU.

Example:If in a certain language, COUNSEL is coded as BITIRAK, how is GUIDANCE written in that code?

Logic: The letters at odd positions are each moved one step backward, while the letters at even positions are respectively moved six, five, four, three, two and so on steps backward to obtain the

corresponding letters of the code.So, the code for the given word GUIDANCE isFOHYZJBFOHYZJBBB.

7.1 Number/Symbol Coding

In this type of Coding-decoding either numeral/symbols code values assigned to the word or alphabetical code letters assigned to the number/symbols.The candidate has to observe

the direction of solving the problem.

Example:If in a certain code, TWENTY is written as 863985 and ELEVEN is written as 323039, how is TWELVE written in that code?

Logic: The alphabets are coded as shown:

T	W	E	N	Y	L	V
8	6	3	9	5	2	0

So, In TWELVE, T is coded as 8,W as 6, E as 3, L as 2 andV as 0. Thus, the code for TWELVE is 863203.

Example:If BRAND is written as 79643 and PAROT is written as 26951, how is PANT coded?

Logic: The alphabets are coded as shown:

B	R	A	N	D	P	O	T
7	9	6	4	3	2	5	1

So, in PANT, P as 2, A as 6, N as 4 and T as 1. Thus, the code for PANT is 2641.

Example:In a certain code, if LOGIC is coded as 1512201824, how isPEARL coded as?

Logic: Each letter's reverse alphabet number. So, PEARL is codedas 112226915.

Example:If APPLE is written as 24991320, how is LOVELY coded as?

Logic: '2' is subtracted from the reverse alphabet numbers of the corresponding letters. Hence, LOVELY is coded as 1310320130.

Example:If A = 2, M = 26, Z = 52, then BET =?

Logic: each letter is assigned a numerical value which is twice the numeral denoting its position in the English alphabet.

B, E and T are 2nd, 5th, and 20th letters respectively.

So, BET = B + E + T = (2 × 2) + (5 × 2) + (20 × 2) = 54.

Example:If in a certain code, BAT = 23 and CAT = 24, then how will you code BALL?

Logic: Taking A=1, B=2, C=3, D=4, ………………, X=24, Y=25, Z=26.

We have: BAT= B+A+T= 2+1+20= 23 and CAT= C+A+T= 3+1+20= 24

So, BALL= B+A+L+L= 2+1+12+12= 27.

Example:If GO = 32, SHE = 49, then SOME will be equal to

Logic: In the given code, Z = 1, Y = 2, X = 3,…………...,C = 24, B = 25, Z = 26.

So, GO = 20 + 12 = 32 and SHE = 8 + 19 + 22 = 49.

Similarly, SOME = S + O + M + E = 8 + 12 + 14 + 22 = 56.

Example:If AT = 20, BAT = 40, then CAT will be equal to

Logic: Taking A = 1, B = 2,………,T = 20,………..., Z = 26, we have:

AT = A × T = 1 × 20 = 20, BAT = B × A × T = 2 × 1 × 20 = 40.

So, CAT = C × A × T = 3 × 1 × 20 = 60.

Example:In a certain code 'CART' is written as '$ #! &' and PLACE is written as '* @ # $ %' How can 'CARE' be written in that code?

Logic: From the data we have C as $, A as #, R as !, T as V, P as *, L as @ and E as %.

So, the code for CARE is $#! %.

Example:In a certain code, RAID is written as %#*$, RIPE is written as % * @!. How is DEAR written up in that code?

Logic: The alphabets are coded as shown:

R	A	I	D	P	E
%	#	*	$	@	!

So, in DEAR, D as $, E as !, A as # and R as %. Thus, the code for DEAR is $! #%

7.2 Substitution

In substitution coding one word is coded with another word. You have to find the right answer for the question and the given code for that word. That will be the right answer for the question.

Example:If ‘cook’ is called ‘butler’, ‘butler’ is called ‘manager’, ‘manager’ is called ‘teacher’, ‘teacher’ is called ‘clerk’ and ‘clerk’ is called ‘principal’, who will teach in a class?

Logic: Teacher teaches in the class and teacher is coded as Clerk so the cleek will be the write answer for the question.

Example:If ‘diamond’ is called ‘gold’, ‘gold’ is called ‘silver’, ‘silver’ is called ‘ruby’ and ‘ruby’ is called ‘emerald’, which is the cheapest jewel?

Logic: The cheapest jewel in the question is silver and silver is coded as the Ruby so ruby will be the right answer.

Example:If white is called blue, blue is called red, red is called yellow, yellow is called green, green is called black, black is called violet and violet is called orange, what would be the color of human blood?

Logic: The colour of the human blood is 'red' but ‘red' is called'yellow'. So, the color of human blood is 'yellow'.

Example:If the animals which can walk are called swimmers, animals who crawl are called flying, those living in water are called snakes and those which fly in the sky are called hunters, then what will a lizard be called?

Logic: Clearly, a lizard crawls and the animals that crawl are called 'flying’. So, 'lizard' is called 'flying'.

7.3 Matrix Coding

Matrix is a set of elements laid out in tabular form (in rows and columns).

Matrix coding is a method to represent the letters of English alphabet by two digits.One digit is represented by the corresponding row and the other digit is represented by the corresponding column.

In Matrix coding problems of logical reasoning, Normally, a letter from these matrices can be represented first by its row and next by its column. A word is represented by only one set of numbers given in any one of the alternatives.

Example:A word is represented by only one set of numbers as given in any one of the alternatives. The set of numbers given in the alternatives are represented by two classes of alphabets as in two matrices given below. The columns and rows of Matrix I are numbered from 0 to 4 and that Matrix II are numbered from 5 to 9. A letter from these matrices can be represented first by its row and next by its column, e.g., ‘A’ can be represented by 01, 14, etc. and E can be represented by 55, 66 etc. Similarly, you have to identify the set for the word ‘BEST’.

Matrix I

	0	1	2	3	4
0	P	A	G	T	S
1	G	T	S	P	A

2	S	P	A	G	T
3	A	G	T	S	P
4	T	S	P	A	G

Matrix II

	5	6	7	8	9
5	E	M	B	N	O
6	B	E	O	M	N
7	O	N	E	B	M
8	N	O	M	E	B
9	M	B	N	O	E

(A) 78, 99,04, 10 (B) 57, 75, 41, 03 (C) 96, 88, 12, 40 (D) 89, 55, 31, 32

Logic: Here, taking the row and column number of each letter, we have

B = 57, 65, 78, 89, 96

E = 55, 66, 77, 88, 99

S = 04, 12, 20, 33, 41

T = 03, 11, 24, 32, 40

Now, taking each option and comparing them with above values, we getonly option (C) contains all the correct values.

So, the code for 'BEST' as 96, 88, 12, 40

7.4 <u>Mixed Letter Coding</u>

In this type of coding and decoding a few statements consisting of the same words but in different order will be coded as words or letters. Candidates need to find the codes of the words by finding the common words in different statements as the code of a word in both the statements will be the same.

Tips to solve The Question

- Take two sentences at a timeand try to find the common words.
- Common words will have common codes in bothsentences.

Example:In a certain code language'pit darna' means 'you are good';'dartok pa' means 'good and bad';'timnatok' means 'they are bad'.In that language, which word stands for 'they'?

Logic:

1. 'Pitdarna' means 'you are good'.
2. 'dartok pa' means 'good and bad'.
3. 'timnatok' means 'they are bad'.

In sentence 1 and 2 'dar' and 'good' are common, therefore 'good' is code for 'dar'.

In sentence 2 and 3 'tok' and 'bad' are common, therefore 'bad' is code for 'tok'.

In sentence 1and 3 'na' and 'are'iscommon, therefore 'are' is code for 'na'.

Now in 3rd sentence, we know the code for 'na' and 'tok', therefore code of 'tim' is 'they'.

Example:In a certain code language, 'col tip mot' means 'singing isappreciable ', 'mot baj min' means 'dancing is good' and 'tip nopbaj' means'singing and dancing', then, which of the following means 'good' in thatcode language?

Logic: In the first and second statements, the common code word is 'mot' and the common word is 'is'. So, 'mot' means 'is'.

In the second and third statements, the common code word is 'baj' and the common word is 'dancing'. So, 'baj' means 'dancing'.

Thus, in the second statements, 'mint' means 'good'.

7.5 Mixed Number Coding

Mixed number coding is the same as mixed letter coding but instead of alphabetical codesnumerical codes are given.

Example:In a certain code language, '851' means 'goodsweet fruit ', '783'means 'good red rose' and '341' means 'rose and fruit'. Which of the following digits stands for 'sweet' in that language?

Logic: In the first and second statements, the common code digit is '8' and the common word is 'good'. So, '8' means 'good'.

In the first and third statements, the common code digit is '1' and the common word is 'fruit'.

So, '1' means 'fruit'.

Thus, in the first statement, '5' means 'sweet'.

Example:In a certain code, '467' means 'leaves are green'; '485' means 'green is good' and '639' means 'they are playing'. Which digit stands for 'leaves' in that code?

Logic: In the first and second statements, the common code digit is '4' and the common word is 'green'. So, '4' means 'green'.

In the first and third statements, the common code digit is '6' and the common word is 'are'. So, '6' means 'are'.

Thus, in the first statement, '7' means 'leaves'.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about different types of Coding -Decodingstatements.
- We have learnt logics to solve different types of Coding-Decoding problems
- We have learnt about some set of rules to solve Coding-Decoding problems

Keywords

- Letter coding
- Number coding
- Substitution coding
- Matrix coding
- Mixed Letter coding
- Mixed number coding

Self Assessment

1. If BROWN is written as 'ZPMUL', then VIOLET is coded as
A. TGMJCR
B. SGMJCQ
C. TGMJCQ
D. TGWCQ

2. If RETURN is coded as PGRWPP, how is SANDLE coded as?
A. UXPBNC
B. QCLFJG
C. CNBPXU
D. CLGFQB

3. In a code language PROPER is coded as NTMRCT, how is RETURN coded in the same code?
A. TCVSTL
B. PGRWPP
C. CNBPXU
D. PGRWQB

4. If E=22, CAT =57 then RAM is equal to
A. 35
B. 30
C. 32
D. 49

5. DEER = 12215 and HIGH = 5654, how will you code HEEL?
A. 2328
B. 3449
C. 4337
D. 5229

6. If ZIP = 198 and ZAP = 246, then how will you code VIP?
A. 174
B. 222
C. 888
D. 990

7. In a certain code 'LAKE' is written as $ #! &and TESLA is written as @&%$# How can 'TASK' be written in that code?
A. #@$!
B. @%#$
C. @#%!
D. None

8. In a code language
'lol sip see' means 'reading is habit ', 'see majrin' means 'playing is good ', 'sip cop maj' means 'reading and playing '.Then which of the following means 'good' in that code language?
A. see

B. rin
C. cop
D. maj

9. If in a code language
'ritpucbec' means 'eat fresh food', 'puc tec jac' means 'food is tasty','jac lac mac' means 'she is beautiful',then which word means 'tasty'?
A. puc
B. bec
C. rit
D. tec

10. In a certain code, '154' means 'Ram is good'; '537' means 'shyamis bad' and '472' means 'good and bad'. Which of the following represents 'shyam'in that code ?
A. 5
B. 1
C. 3
D. 2

11.If blue is called red, red is called green, green is called black and black is called white, what is the colour of sky?
A. red
B. black
C. white
D. None of these

12. If SYMBOL is written as NZTMPC is a certain code. How is NUMBER written in that code?
A. NVOSFC
B. OVSFSC
C. NVOFCS
D. None

13.If AC is called TV, TV is called fridge, fridge is called washing machine, washing machine is called computer, computer is called phone, phone is called printer and printer is called calculator then what would be used to wash clothes?
A. TV
B. Calculator
C. Computer
D. Printer

14. In a code language if SEVEN is written as 19522514, then in the same code language how EIGHT will be written.
A. 597820
B. 598718
C. 598207
D. 592087

15. A word is represented by only one set of numbers as given in any one of the alternatives.The sets of numbers given in the alternatives are represented by two classes of alphabets as in

two matrices given below. The columns and rows of Matrix I are numbered – from 0 to 4 and that Matrix II are numbered from 5 to 9. A letter from these matrices can berepresented first by its row and next by its column, e.g., 'U' can be represented by 01, 14, etc. and E can be represented by 55, 66 etc. Similarly, you have to identify the set for the word 'JUDGE'.

Matrix I

	0	1	2	3	4
0	J	U	G	R	Z
1	G	R	Z	J	U
2	Z	J	U	G	R
3	U	G	R	Z	J
4	R	Z	J	U	G

Matrix II

	5	6	7	8	9
5	E	M	D	N	O
6	D	E	O	M	N
7	O	N	E	D	M
8	N	O	M	E	D
9	M	D	N	O	E

A. 13, 31, 96, 10, 88
B. 00, 30, 56, 31, 99
C. 42, 43, 65, 21, 55
D. 34, 01, 89, 23, 66

Answers for SelfAssessment

l.	A	2.	B	3.	B	4.	D	5.	D
6.	B	7.	C	8.	B	9.	D	10.	C
11.	A	12.	A	13.	C	14.	A	15.	D

Review Questions

1. In a certain language if HOUSE is written as KRUQC then how ROHIT will be written in that language?
2. If BRAND is written as 79643 and PAROT is written as 26951, how is PANT coded?
3. If SUNDAY is written as NYNYNY, how will BANGALORE be written in that code?
4. If EARTH is written as FCUXM in a certain code. How is MOON written in that code?
5. In a certain code language, '123' means 'hot filtered coffee', '356'means 'very hot day' and '589' means 'day and night'. Which digit stands for'very'?

Further Reading

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, S Chand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Web Links

1. https://www.examveda.com/mcq-question-on-competitive-reasoning/
2. https://www.hitbullseye.com/Reasoning

Unit 08: Simple Interest

CONTENTS

Objectives

After studying this unit, you will be able to

- define the concept of principal, rate, time, simple interest, and amount.
- explore the formulae of principal, rate, time, simple interest, and amount.
- analyze effect of change of principal, rate, and time on simple interest.
- solve problems based on simple interest computation, principal computation, rate, time, and amount computation.

Introduction

Problem related to computation of principal, rate, time, simple interest, and amountis asked in almost all competitive exams. In this chapter you will be expose to the concept of principal, rate, time, simple interest, and amount; effect of change of principal, rate, and time on simple interest; and procedure to compute principal, rate, time, simple interest, and amount based on the given information.

8.1 Basics of Principal, Rate and Time

The basics of principal, rate, and time are explained below:

Principal

Principal is the total amount of money borrowed by an individual from other individual.

Principal is the money borrowed or lent out for a certain period.

The principal is the amount that initially borrowed from the bank or invested.

It is generally denoted by ‘P’ or ‘p’.

Rate/Rate of Interest

Interest is the extra money paid for using other's money. It is interest per cent. For example 5% or 9%.

The interest is usually charged according to a specified term, which is expressed as some per cent of the principal and is called the rate of interest for a fixed period.It is generally denoted by 'R' or 'r'

Fixed Period may be a year, six months, three months, a month, etc.

Rate of interest may be calculated annually, semi-annually, quarterly, monthly etc.

R % or r% in simple form are written as $\frac{R}{100}$and $\frac{r}{100}$.

For example

- Rate of interest - 5%, 10%, or 12%, etc.
- Rate of interest is 5% per annum.
- Interest payable on 100 for one year is 5.

Time

It is defined as the duration for which the principal amount is given to someone. For example, a year, six months, three months, a month etc. It is generally denoted by 'T' and 't'.

Simple Interest

It is the interest on a sum borrowed for a certain period is reckoned uniformly.

It is that interest which is payable on the principal amount only.

The general abbreviation of simple interest is 'S.I.' or 'I'.

It is the interest computed on the principal amount for the entire period it is borrowed.

For example, simple interest on Rs. 100 at 5% per annum = Rs. 5 each year.

Amount

Amount is defined as the principal borrowed plus the interest occurred on principal.

Formulae

- Simple Interest = $\frac{\text{Principal x Rate x Time}}{100}$ or S.I. = $\frac{P \times R \times T}{100}$ or I = $\frac{p \times r \times t}{100}$
- Principal = $\frac{\text{Simple Interest x 100}}{\text{Rate x Time}}$ or P = $\frac{S.I. \times 100}{R \times T}$or p = $\frac{I \times 100}{r \times t}$
- Rate = $\frac{\text{Simple Interest x 100}}{\text{Principal x Time}}$ or R = $\frac{S.I. \times 100}{P \times T}$ or r = $\frac{I \times 100}{p \times t}$
- Time = $\frac{\text{Simple Interest x 100}}{\text{Principal xRate}}$or T = $\frac{S.I. \times 100}{P \times R}$ or t = $\frac{I \times 100}{p \times r}$
- Amount = Principal + Simple Interest/Interest or A = P (or p) + S.I. (or I)
- Principal = Amount – Interest or P = A – S.I.
- Interest = Amount – Principal or S.I.= A – P
- Amount = Principal + $\frac{\text{Principal x Rate x Time}}{100}$
- Amount = Principal (1 + $\frac{\text{Rate x Time}}{100}$) orA = P (1 + $\frac{R \times T}{100}$) or A = p (1 + $\frac{r \times t}{100}$)

8.2 Computation of Simple Interest

Example 1

Calculate the simple interest on Rs. 3000 for 3 years at 5% per annum.

Solution – Given,

P = Rs. 3000, R = 5% p.a., and T = 3 years

We know that S.I. = $\frac{P \times R \times T}{100}$

S.I. = $\frac{3000 \times 5 \times 3}{100}$

S.I. = Rs. 450

Example 2

Find the simple interest on Rs. 32000 at $7\frac{1}{2}$ % p.a. for 8 months.

Solution – Given,

P = Rs. 32000, $R = 7\frac{1}{2}$ % p.a. = $\frac{15}{2}$ % p.a., & T = 8 months = $\frac{8}{12}$ years = $\frac{2}{3}$ years

We know that S.I. = $\frac{P \times R \times T}{100}$

S.I. = $\frac{32000 \times 15 \times 2}{100 \times 2 \times 3}$

S.I. = Rs. 1600

Example 3

Find the simple interest on Rs 5000 at 15 % p.a. for the period from 25thMay, 2022 to 28th July, 2022.

Solution – Given,

P = Rs. 5000 &R = 15 % p.a.

T = (6 days of May + 30 days of June + 28 days of July)

T = 64 days=$\frac{64}{365}$ years

We know that S.I. = $\frac{P \times R \times T}{100}$

S.I. = $\frac{5000 \times 15 \times 64}{100 \times 365}$

S.I. = Rs. $\frac{9600}{73}$

S.I. = Rs. 131.51 (approx.)

Example4

The simple interest accrued on an amount of Rs. 1500 at the end of 2 years is Rs. 600. What would be the simple interest accrued on an amount of Rs. 3400 at the same rate and for the same period?

Solution – Given,

P = Rs. 1500, T = 2 years, & S.I. = Rs. 600

We know that $R = \frac{S.I. \times 100}{P \times T}$

$R = \frac{600 \times 100}{1500 \times 2}$

$R = \frac{600 \times 100}{1500 \times 2}$

R = 20%

Again, P = Rs. 3400, R = 20 % p.a., & T = 2 years

We know that S.I. = $\frac{P \times R \times T}{100}$

S.I. = $\frac{3400 \times 20 \times 2}{100}$

S.I. = Rs. 1360

8.3 Effect of Change of P, R & T on S.I.

Change in Simple Interest (S.I.)= $\frac{[\text{Product of Fixed Parameter}]\times[\text{Difference of Product of Variable Parameters}]}{100}$

Did you know?

- If P changes but R & T remains same, then change in S.I. = $\frac{[R\times T] \times [\text{Change in P}]}{100}$
- If R changes but P & T remains same, then change in S.I. = $\frac{[P\times T]\times [\text{Change in R}]}{100}$
- If T changes but P & R remains same, then change in S.I. = $\frac{[P\times R]\times [\text{Change in T}]}{100}$
- If R changes from R_1 to R_2, T changes from T_1 to T_2 , and P is fixed, then

  Change in S.I. = $\frac{P\times [R_1T_1 - R_2T_2]}{100}$
- If P changes from P_1 to P_2, T changes from T_1 to T_2 , and R is fixed, then

  Change in S.I. = $\frac{R\times [P_1T_1 - P_2T_2]}{100}$
- If P changes from P_1 to P_2, R changes from R_1 to R_2 , and T is fixed, then

  Change in S.I. = $\frac{T\times [P_1R_1 - P_2R_2]}{100}$

Example 5

Calculate the change in simple interest (S.I.) for 3 years at 2% p.a. if the principal increases by Rs. 1000.

Solution – Given,

Change in P = Rs. 1000, R = 2 % p.a., & T = 3 years

We know that Change in S.I. $= \frac{[R \times T]\times [\text{Change in P}]}{100}$

$= \frac{2 \times 3 \times 1000}{100}$

= Rs. 60

Example 6

Calculate the change in simple interest (S.I.) on Rs. 500 for 3 years if rate % increases by 2% p.a.

Solution – Given,

P = Rs. 500, Change in R = 2 % p.a., & T = 3 years

We know that Change in S.I. $= \frac{[P \times T]\times [\text{Change in R}]}{100}$

$= \frac{500 \times 3 \times 2}{100}$

= Rs. 30

Example 7

Calculate the change in simple interest (S.I.) on Rs. 500 at 2% p.a. if the time increases by 6 years.

Solution – Given,

P = Rs. 500, R = 2 % p.a., & Change in T = 6 years

We know that Change in S.I. $= \frac{[P \times R] x [\text{Change in } T]}{100}$

$= \frac{500 \times 2 \times 6}{100}$

= Rs. 60

Example 8

Calculate the change in simple interest (S.I.) on Rs. 8000 if the rate % changes from 4% p.a. to 5% p.a. and the time changes from 4 years to 3 years.

Solution – Given,

P = Rs. 8000, R_1 = 4 % p.a., R_2 = 5 % p.a., T_1 = 4 yrs & T_2 = 3 yrs

We know that Change in S.I. $= \frac{P \times [R_1 T_1 - R_2 T_2]}{100}$

$= \frac{8000 \times [4 \times 4 - 5 \times 3]}{100}$

= 80 x 1

= Rs. 80

8.4 Computation of Principal

Example 9

A man earns Rs. 450 as interest in 3 years on a certain money invested in a company at the rate of 5% p.a. Calculate the principal invested by the man in the company.

Solution – Given,

S.I. = Rs. 450, R = 5% p.a., andT = 3 years

P =?

We know that $P = \frac{S.I. \times 100}{R \times T}$

$P = \frac{450 \times 100}{5 \times 3}$

P = Rs. 3000

Example 10

What principal will amount to Rs. 2500 at 2% p.a. in $12\frac{1}{2}$ years.

Solution – Given,

A = Rs. 2500, R = 2% p.a., and T = $12\frac{1}{2}$ years = $\frac{25}{2}$ years

P =?

We know that $P = \frac{A \times 100}{100 + R \times T}$

$P = \frac{2500 \times 100}{100 + 2 \times \frac{25}{2}}$

$P = \frac{2500 \times 100}{100 + 25}$

$P = \frac{250000}{125}$

P = Rs. 2000

Example 11

Calculate the annual instalment that will discharge a debt of Rs.6500 due in 5 years at 3% p.a. simple interest.

Solution – Given

A = Rs. 6500, R = 3% p.a., and T = 5 years

Annual instalment =?

We know that $P = \frac{A \times 100}{100 \times T + \frac{R \times T \times (T-1)}{2}}$

$$P = \frac{6500 \times 100}{100 \times 5 + \frac{3 \times 5 \times (5-1)}{2}}$$

$$P = \frac{6500 \times 100}{100 + \frac{15 \times (4)}{2}}$$

$$P = \frac{6500 \times 100}{100 + 30}$$

$$P = \frac{650000}{130}$$

P = Rs. 5000

Example 12

Calculate principal if it amounts to Rs. 5000 in 2 years and to Rs. 6000 in 3 years at simple interest.

Solution – Given,

A_1=Rs.5000, A_2=Rs.6000, T_1=2 years,and T_2=3 years

P = ?

We know that $P = \frac{[A_1T_2 - A_2T_1]}{[T_2 - T_1]}$

$$= \frac{[5000 \times 3 - 6000 \times 2]}{[3 - 2]}$$

$$= \frac{[3000]}{[1]}$$

= Rs. 3000

Example 13

Calculate the sum if it amounts to Rs. 500 at 8% p.a. and amounts to Rs. 400 at 4% p.a.

Solution – Given,

A_1=Rs. 500, A_2=Rs. 400, R_1 = 8% p.a., andR_2 =4% p.a.

P =?

We know that $P = \frac{[A_2R_1 - A_1R_2]}{[R_1 - R_2]}$

$$= \frac{[400 \times 8 - 500 \times 4]}{[8 - 4]}$$

$$= \frac{[3200 - 2000]}{[4]}$$

$$= \frac{[1200]}{[4]}$$

= Rs. 300

Example 14

What will be the original sum of money if annual income is Rs. 2400, $\frac{1}{2}$of it is invested at 1%, $\frac{1}{4}$at 3% and the rest at 5%.

Solution – Given,

A = 2400, R_1= 1% p.a., R_2= 3% p.a., and R_3= 5% p.a.

P =?

Here, $\frac{1}{a} = \frac{1}{2}$, $\frac{1}{b} = \frac{1}{4}$

$$\frac{1}{c} = 1 - (\frac{1}{a} + \frac{1}{b}) = 1 - (\frac{1}{2} + \frac{1}{4})$$

$$\frac{1}{c} = 1 - (\frac{2+1}{4})$$

$$\frac{1}{c} = 1 - (\frac{3}{4}) = (\frac{4-3}{4})$$

$$\frac{1}{c} = (\frac{1}{4})$$

Now A =Rs. 2400, R_1 = 1% p.a., R_2 = 3% p.a., R_3= 5% p.a., and P =?

a = 2, b = 4 , c = 4

Original Sum of Money (P) $= \frac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}}$

$$= \frac{2400 \times 100}{\frac{1}{2} + \frac{3}{4} + \frac{5}{4}}$$

$$= \frac{240000}{\frac{2+3+5}{4}}$$

$$= \frac{240000}{\frac{10}{4}}$$

$$= \frac{240000 \times 4}{10}$$

= Rs.96000

Example 15

Calculate the sum if the simple interest on a certain sum of money for 6 years at 10% p.a. is Rs. 30 less than the simple interest on the same sum for 5 years at 8% p.a.

Solution – Given,

Difference in S.I. = Rs. 30, R_1= 10% p.a., T_1= 6 years, T_2= 5 years, and R_2= 8% p.a.

P =?

We know that S. I. $= \frac{P \times R \times T}{100}$

Difference in S.I. $= [\frac{P \times R_1 \times T_1}{100} - \frac{P \times R_2 \times T_2}{100}]$

$$= P[\frac{R_1 \times T_1}{100} - \frac{R_2 \times T_2}{100}]$$

$$P = \frac{\text{Difference in S.I.}}{[\frac{R_1 \times T_1}{100} - \frac{R_2 \times T_2}{100}]}$$

$$P = \frac{30}{[\frac{10x6}{100} - \frac{8x5}{100}]}$$

$$P = \frac{30}{\left[\frac{60 - 40}{100}\right]}$$

$$P = \frac{30 \times 100}{[20]}$$

$$P = \text{Rs. } 150$$

8.5 Computation of Rate

Example 16

At what interest rate p.a. the simple interest accrued on an amount of Rs. 1500 at the end of 2 years is Rs. 600?

Solution – Given,

P = Rs. 1500, T = 2 years, and S.I. = Rs. 600

R =?

We know that $R = \frac{\text{S.I.} \times 100}{P \times T}$

$$R = \frac{600 \times 100}{1500 \times 2}$$

$$R = 20\%$$

Example 17

At what interest rate p.a., in 4 years, a sum of Rs. 2000 will become Rs. 4000?

Solution – Given,

P = Rs. 2000, A = Rs. 4000, T = 4 years

R =? and S.I. =?

We know that S.I. = A − P

S.I. = Rs. (4000 − 2000)

S.I.= Rs. 2000

We know that $R = \frac{\text{S.I.} \times 100}{P \times T}$

$$R = \frac{2000 \times 100}{2000 \times 4}$$

R= 25 % p.a.

Example 18

Calculate rate per cent p.a. if a sum of money trebles (increase three times) itself in 4 years simple interest.

Solution – Given,

n = 3 and T = 4 years

R =?

We know that $R = \frac{(n-1) \times 100}{T}$

$R = \frac{(3-1) \times 100}{4}$

$R = \frac{(2) \times 100}{4}$

$R = 50 \%$ p.a.

Example 19

Calculate rate of interest if a sum of money at simple interest amounts to Rs. 5000 in 2 years and to Rs. 6000 in 3 years.

Solution – Given,

A_1=Rs.5000, A_2= Rs.6000, T_1= 2 years, T_2=3 years, andR = ?

We know that $R = \frac{[A_2 - A_1]}{[A_2T_1 - A_1T_2]}$

$= \frac{[6000 - 5000]}{[5000 \times 3 - 6000 \times 2]}$

$= \frac{[1000]}{[15000 - 12000]}$

$= \frac{[1000]}{[3000]}$

$= \frac{1}{3} \%$

Example 20

A person 'X' deposits Rs. 1000 and Rs. 500 in saving at 3.5% p.a. and 5% p.a. respectively. Compute the rate of interest for the whole sum.

Solution – Given,

P_1= Rs. 1000, P_2= Rs. 500, R_1= 3.5% p.a., and R_2= 5% p.a.

R = ?

We know that R $= \frac{[P_1R_1 + P_2R_2]}{[P_1 + P_2]}$

$= \frac{[1000 \times 3.5 + 500 \times 5]}{[1000 + 500]}$

$= \frac{[6000]}{[1500]}$

$= 4 \%$ p.a.

8.6 Computation of Time

Example 21

In what time Rs.20000 will earn an interest of Rs.2000 at 8% p.a.?

Solution – Given,

P = Rs. 20000, S.I. = Rs. 2000, and R = 8% p.a.

T =?

We know that $T = \frac{S.I. \times 100}{P \times R}$

$T = \frac{S.I. \times 100}{P \times R}$

$T = \frac{2000 \times 100}{20000 \times 8}$

$T = 1\frac{1}{4}$ years

T = 1.25 years

T = 1 year and 3 months

Example 22

In what time a sum of money will four times itself at a rate of simple interest of 12% p.a.?

Solution – Given,

n = 4 and R = 12 % p.a.

T =?

We know that $T = \frac{(n-1) \times 100}{R}$

$T = \frac{(4-1) \times 100}{12}$

$T = \frac{(4-1) \times 100}{12}$

$T = \frac{(3) \times 100}{12}$

T = 25 years

Example 23

In how many years will a sum of money double itself at $13\frac{1}{2}$% p.a.?

Solution – Given,

n = 2 and $R = 13\frac{1}{2} = \frac{27}{2}$ %

T =?

We know that $T = \frac{(n-1) \times 100}{R}$

$T = \frac{(2-1) \times 100}{\frac{27}{2}}$

$T = \frac{(1) \times 100 \times 2}{27}$

$T = \frac{200}{27}$

$T = 7\frac{11}{27}$years

Example 24

A sum of money put out on simple interest doubles itself in 10 years. In how many years would it five times itself?

Solution – Given,

n = 2, m = 5, and T = 10 years

We know that Required Time (T') $= \frac{(m-1) \times T}{(n-1)}$

$= \frac{(5-1) \times 10}{(2-1)}$

$= \frac{(4) \times 10}{(1)}$

= 40 years

Example 25

A certain sum of money becomes three times of itself in 20years at simple interest. In how many years does it become double of itself at the same rate of simple interest?

Solution – Given,

n = 3, m = 2, and T = 20 years

Required Time (T') $= \frac{(m-1) \times T}{(n-1)}$

$= \frac{(2-1) \times 20}{(3-1)}$

$= \frac{(1) \times 20}{(2)}$

= 10 years

Example 26

Calculate the time if a sum amounts to Rs. 500 at 8% p.a. and amounts to Rs. 400 at 4% p.a.

Solution – Given,

$A_1 = 500, A_2 = 400, R_1 = 8\%$ p.a.and $R_2 = 4\%$ p.a.

T = ?

We know that T $= \frac{[A_1 - A_2]}{[A_2R_1 - A_1R_2]}$

$= \frac{[500 - 400]}{[400 \times 8 - 500 \times 4]}$

$= \frac{[100]}{[3200 - 2000]}$

$= \frac{[100]}{[1200]}$

$= \frac{1}{12}$ years

= 1 month

8.7 Computation of Amount

Example27

If the interest rate is 3% p.a., then a sum of Rs. 2000 amount to how much in 4 years?

Solution – Given,

P = Rs. 2000, R = 3% p.a., and T = 4 years

A =?

We know that S.I. $= \frac{P \times R \times T}{100}$

S.I. $= \frac{2000 \times 3 \times 4}{100}$

S.I. = Rs. 240

A = P + S.I.

A = Rs. (2000 + 240)

A = Rs. 2240

Example28

A person borrowed Rs. 7000 from his friend at 5% p.a. for 2years. Find the money returned by that person to his friend.

Solution – Given,

P = Rs. 7000, R = 5% p.a., and T = 2 years

A =?

We know that $A = P\left(1 + \frac{R \times T}{100}\right)$

$$A = 7000\left(1 + \frac{5 \times 2}{100}\right)$$

$$A = 7000\left(1 + \frac{1}{10}\right)$$

$$A = 7000\left(\frac{10+1}{10}\right)$$

$$A = 7000\left(\frac{10+1}{10}\right)$$

$$A = 7000\left(\frac{11}{10}\right)$$

A = Rs. 7700

Summary

In short,

- Simple Interest = $\frac{\text{Principal x Rate x Time}}{100}$
- Amount = Principal + Interest
- $P = \frac{S.I. \times 100}{R \times T}$
- If a certain sum in T years at R% per annum amounts to Rs. A, then

  $P = \frac{A \times 100}{100 + R \times T}$
- The annual payment that will discharge a debt of Rs. A due in T years at R% per annum is $P = \frac{A \times 100}{100 \times T + \frac{R \times T \times (T-1)}{2}}$
- If a certain sum of money P lent out at S.I. amounts to A_1 in T_1 years and to A_2 in T_2 years, then $P = \frac{[A_1T_2 - A_2T_1]}{[T_2 - T_1]}$
- If a certain sum of money P lent out for a certain time T amounts to A_1 at R_1% per annum and to A_2 at R_2% per annum, then $P = \frac{[A_2R_1 - A_1R_2]}{[R_1 - R_2]}$
- Original Sum of Money (P)= $\frac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}}$
- $P = \frac{\text{Difference in S.I.}}{\left[\frac{R_1 \times T_1}{100} - \frac{R_2 \times T_2}{100}\right]}$
- $R = \frac{S.I. \times 100}{P \times T}$

- If a certain sum of money becomes n times itself in T years at simple interest, then the rate of interest per annum is $R = \frac{(n-1) \times 100}{T}$
- If a certain sum of money P lent out at S.I. amounts to A_1 in T_1 years and to A_2 in T_2 years, then R $= \frac{[A_2 - A_1]}{[A_2T_1 - A_1T_2]}$
- If an amount P_1 lent at simple interest rate of R_1% per annum, and another amount P_2 at simple interest rate of R_2 % per annum, then the rate of interest for the whole sum is $R = \frac{[P_1R_1 + P_2R_2]}{[P_1 + P_2]}$
- $T = \frac{S.I. \times 100}{P \times R}$
- If a certain sum of money becomes n times itself at R% p.a. simple interest in T years, thenT = $\frac{(n-1) \times 100}{R}$
- If a certain sum of money becomes n times itself in T years at a simple interest, then the time (T') in which it will become m times itself is given by Required Time (T') $= \frac{(m-1) \times T}{(n-1)}$
- If a certain sum of money P lent out for a certain time T amounts to A_1 at R_1% p.a. and to A_2 at R_2% p.a., thenT $= \frac{[A_1 - A_2]}{[A_2R_1 - A_1R_2]}$
- A = P + S.I.
- A = $P(1 + \frac{R \times T}{100})$

Keywords

Principalis the total amount of money borrowed.

Rate or Rate of Interestis the extra money paid per cent.

Timeis the duration for which the principal amount is given.

Simple Interestis the interest computed on the principal amount for the entire period it is borrowed.

Amountis theprincipal borrowed plus the interest.

Self Assessment

1. The simple interest on Rs. 500 at 6% per annum from May 3rd to July 15th in the same year is:

A. Rs. 9
B. Rs. 6
C. Rs. 4
D. Rs. 5

2. What would be the simple interest obtained on an amount of ` 5760 at the rate of 6 p.c.p.a. after 3 years.?

A. Rs. 1036.80
B. Rs. 1063.80
C. Rs. 1336.80
D. Rs. 1666.80

3. A man borrowed a sum of Rs. 10000 from a finance company for 6 years at 8% per annum. The amount returned by man to the finance company is

A. Rs. 14800
B. Rs. 12600
C. Rs. 13300
D. Rs. 12040

4. A farmer borrowed Rs. 3600 at 15% simple interest per annum. At the end of 4 years, he cleared this account by paying Rs. 4000 and a cow. The cost of the cow is
A. Rs. 1000
B. Rs. 1200
C. Rs. 1550
D. Rs.1760

5. The principal that will yield Rs. 60 as simple interest at 6% per annum in 5 years is
A. Rs. 175
B. Rs. 350
C. Rs. 200
D. Rs. 259

6. 'X' borrows Rs. 520 from 'Y' at a simple interest of 13% per annum. What amount of money should 'X' pay to 'Y' after 6 months to be absolved of the debt?
A. Rs. 353.80
B. Rs. 453.80
C. Rs.552.80
D. Rs. 553.80

7. The sum of money that will produce Rs. 1770 interest in $7\frac{1}{2}$ years at 8% simple interest per annum is
A. Rs. 2950
B. Rs. 3120
C. Rs. 2800
D. Rs. 1359

8. A sum fetched a total simple interest of Rs. 4016.25 at the rate of 9 p.c. p.a. in 5 years. What is the sum?
A. Rs. 4462.50
B. Rs. 8032.50
C. Rs. 8900
D. Rs. 8925

9. If the simple interest on a certain sum of money after $6\frac{1}{4}$ years is $\frac{3}{8}$ of the principal, then the rate of interest per annum is
A. 5%
B. 6%
C. 4%
D. 7%

10. In 4 years, Rs. 6000 amounts to Rs. 8000. In what time at the same rate will Rs. 525 amount to Rs. 700?
A. 2 years
B. 3 years
C. 4 years
D. 5 years

11. In how many years will a sum of money treble itself at 10% per annum simple interest?
A. 15 years
B. 19 years
C. 20 years
D. 12 years

12. Rs. 6200 amounts to Rs. 9176 in 4 years at simple interest. If the interest rate is increased by 3% it would amount to how much?

A. Rs. 8432

B. Rs. 9820

C. Rs. 9920

D. Rs. 10920

13. A sum of money doubles itself in 8 years. In how many years will it treble?

A. 16 years

B. 15 years

C. 14 years

D. 21 years

14. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2% higherrate, it would have fetched Rs. 56 more. Find the sum.

A. Rs. 680

B. Rs. 700

C. Rs. 720

D. Rs. 530

15. A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?

A. Rs. 35

B. Rs. 245

C. Rs. 350

D. Cannot be determined

Answers for Self Assessment

l.	B	2.	A	3.	A	4.	D	5.	C
6.	D	7.	A	8.	D	9.	B	10.	C
11.	C	12.	C	13.	A	14.	D	15.	B

Review Questions

16. If the simple interest on Rs. 1400 be more than the interest on Rs. 1000 by Rs. 60 in 5 years, then find the out the rate per cent per annum.
17. A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But, when invested at 5% per annum, it amounts to `350. Find out the sum and time.
18. What is the present worth of Rs.132 due in 2 years at 5% simple interest per annum?

19. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year?
20. Out of a certain sum, $\frac{1}{3}$ is invested at 3%, $\frac{1}{6}$ at 6% and the rest at 8%. If the annual income is Rs. 300, then find original sum.
21. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum?
22. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is Rs. 840. At what rate of interest the same amount of interest can be received on the same sum after 5 years?
23. If the simple interest on `1400 be more than the interest on `1000 by `60 in 5 years, fi nd the out the rate per cent per annum.
24. A sum of money put out on simple interest doubles itself in 12 $\frac{1}{2}$ years. In how many years would it treble itself?
25. Rahul deposits Rs. 5000 in NSC at 2% per annum and Rs. 2000 in mutual funds at 4% per annum. Find out the rate of interest for the whole sum.

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
2. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal. S. Chand & Co Ltd. (2010).
3. Magical Book on Quicker Mathsby M Tyra, Banking Service Chronicle.
4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle.
5. Quantitative Aptitude for Competitive Examinations by Dinesh Khattar, Pearson Education (2020).

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 09: Compound Interest

CONTENTS

Objectives

After studying this unit, you will be able to

- define the concept of compound interest.
- explore the formulae of compound interest.
- differentiate between simple and compound interest.
- solve problems based on compound interest computation.
- solve problems based on principal and rate computation.
- solve problems based on time and amount computation.
- solve problems based on relation between compound and simple interest.

Introduction

Problem related to computation of principal, rate, time, compound interest, and amountis asked in almost all competitive exams. In this chapter you will be expose to the concept of compound interest; formulae of compound interest; and procedure to solve problems based oncompound interest,principal, rate, time, amount,and relation between compound and simple interest for given data.

9.1 Compound Interest

When the borrower and lender agree to fix up a specific unit of time i.e., yearly/half-yearly/quarterly to settle the previous account, then the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

In this situation, the difference between the money borrowed and amount is called the compound interest for that specific period. The general abbreviation used for compound interest is C.I.

Mathematically, Compound Interest = Amount – Principal

Basic Formulae

Let P or p = Principal

R or r = Rate

N or n = Number of years (T or t)

C.I. = Compound interest

A = Amount

- Compound Interest = Amount – Principals
- When interest is compounded annually
  - Amount = P $\left[1+\frac{\text{Rate}}{100}\right]^n$
  - Compound Interest = P $\left[1+\frac{R}{100}\right]^n$ – P
  - Compound Interest = P$\left\{\left[1+\frac{R}{100}\right]^n - 1\right\}$
- Rate of interest
  - $A = P\left[1+\frac{R}{100}\right]^n$
  - $\frac{A}{P} = \left[1+\frac{R}{100}\right]^n$
  - $\left[\frac{A}{P}\right]^{1/n} = \left[1+\frac{R}{100}\right]$
- Rate of interest
  - $\left[\frac{A}{P}\right]^{1/n} - 1 = \frac{R}{100}$
  - $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100 = R$
  - Rate of interest (R) = $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100$

    or $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\}$% p.a.
- When interest is compounded Half-yearly
  - Amount = P $\left[1+\frac{\frac{R}{2}}{100}\right]^{2n}$
  - Compound Interest = P $\left[1+\frac{\frac{R}{2}}{100}\right]^{2n}$ – P
  - Compound Interest = P$\left\{\left[1+\frac{\frac{R}{2}}{100}\right]^{2n} - 1\right\}$
  - Rate of interest (R) = 2 x 100 x$\left\{\left[\frac{A}{P}\right]^{1/2n} - 1\right\}$
- When interest is compounded Quarterly
  - Amount = P $\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}$
  - Compound Interest = P $\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}$ – P
  - Compound Interest = P$\left\{\left[1+\frac{\frac{R}{4}}{100}\right]^{4n} - 1\right\}$
  - Rate of interest (R) = 4 x 100 x $\left\{\left[\frac{A}{P}\right]^{1/4n} - 1\right\}$

- When interest is compounded annually but time is in fraction, $1\frac{1}{2}$ years.
  - Amount = P $\left[1+\frac{R}{100}\right]^1 \times \left[1+\frac{\frac{1}{2}R}{100}\right]$
  - Compound Interest = P$\left[1+\frac{R}{100}\right]^1 \times \left\{1+\frac{\frac{1}{2}R}{100}\right] - P$
  - Compound Interest = P$\left\{\left[1+\frac{R}{100}\right]^1 \times \left[1+\frac{\frac{1}{2}R}{100}\right] - 1\right\}$
- When rates are different for different years
  - R_1 %, R_2 %, R_3 %, R_4 %, …. for 1st, 2nd, 3rd, 4th year …. respectively.
  - Amount =P x$\left[1+\frac{R_1}{100}\right] \times \left[1+\frac{R_2}{100}\right] \times \left[1+\frac{R_3}{100}\right] \times \left[1+\frac{R_4}{100}\right] \times$ …….
- Present worth of Rs. x due n years hence
  - Present Worth = $\frac{x}{\left[1+\frac{R}{100}\right]^n}$
- The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum
  - In terms of P and R
    - C.I. – S.I. = P $\left[\frac{R}{100}\right]^2$
  - In terms of S.I. and R
    - C.I. – S.I. = $\frac{R \times S.I.}{2 \times 100}$

Derivation-

Let, given sum of money = Rs. P

Simple interest on Rs. P for 2 years at R% per annum = $\frac{P \times R \times 2}{100}$

Compound interest on Rs. P for 2 years at R% per annum= P$\left\{\left[1+\frac{R}{100}\right]^2 - 1\right\}$

C.I. – S.I. =P$\left\{\left[1+\frac{R}{100}\right]^2 - 1\right\}$ - $\frac{P \times R \times 2}{100}$

C.I. – S.I. =P$\left\{1+\left[\frac{R}{100}\right]^2 + \frac{2R}{100} - 1- \frac{2R}{100}\right\}$

C.I. – S.I. =P$\left\{\left[\frac{R}{100}\right]^2\right\}$

C.I. – S.I. =P x $\frac{R}{100} \times \left\{\frac{R}{100}\right\}$

C.I. – S.I. = $\frac{R}{100 \times 2} \times \left\{\frac{P \times R \times 2}{100}\right\}$

C.I. – S.I. = $\frac{R \times S.I.}{100 \times 2}$ (Because S.I. = $\frac{P \times R \times 2}{100}$)

- The difference between the compound interest and the simple interest on a certain sum of money for 3 years at R% per annum
  - In terms of P and R
    - C.I. – S.I. =P$\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$
  - In terms of S.I. and R
    - C.I. – S.I. = $\frac{S.I.}{3}\left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

Derivation-

Let, given sum of money = Rs. P

Simple interest on Rs. P for 3 years at R% per annum = $\frac{P \times R \times 3}{100}$

Compound interest on Rs. P for 3 years at R% per annum= $P\left\{\left[1+\frac{R}{100}\right]^3 - 1\right\}$

C.I. – S.I. $= P\left\{\left[1+\frac{R}{100}\right]^3 - 1\right\} - \frac{P \times R \times 3}{100}$

C.I. – S.I. $= P\left\{1+\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2 + \frac{3R}{100} - 1 - \frac{3R}{100}\right\}$

C.I. – S.I. $= P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$

C.I. – S.I. $= P\left[\frac{R}{100}\right]\left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

C.I. – S.I. $= \frac{P \times R \times 3}{100} \times \frac{1}{3} \times \left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

C.I. – S.I. $= S.I. \times \frac{1}{3} \times \left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$ (Because S.I. = $\frac{P \times R \times 2}{100}$)

C.I. – S.I. $= \frac{S.I.}{3} \times \left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

- If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Derivation-

Let the sum of money = Rs. P

$nP = P\left[1+\frac{R}{100}\right]^t$

$n = \left[1+\frac{R}{100}\right]^t$ ………… (1)

Let the sum become n^m times in T years.

$n^m = \left[1+\frac{R}{100}\right]^T$

$n = \left[1+\frac{R}{100}\right]^{T/m}$ ………… (2)

From (1) and (2)

$\left[1+\frac{R}{100}\right]^t = \left[1+\frac{R}{100}\right]^{\frac{T}{m}}$

$t = \frac{T}{m}$

T = mt years

Hence, sum becomes n^m times in mt years.

- If a certain sum becomes n times in t years, then the rate of compound interest is

$R = 100[(n)^{1/t} - 1]$

- If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is $R = \left\{\left[\frac{y}{x}\right]^{1/(B-A)} - 1\right\} \times 100$ %

Derivation -

Let Principal = Rs. P

Rate of interest = R% p.a.

A. T. Q., $x = P\left[1+\frac{R}{100}\right]^A$ and $y = P\left[1+\frac{R}{100}\right]^B$

$\frac{y}{x} = \frac{P\left[1+\frac{R}{100}\right]^B}{P\left[1+\frac{R}{100}\right]^A}$

$\frac{y}{x} = \left[1+\frac{R}{100}\right]^{B-A}$

$$1 + \frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)}$$

$$1 + \frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)}$$

$$\frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)} - 1$$

$$R = \left\{\left[\frac{y}{x}\right]^{1/(B-A)} - 1\right\} \times 100$$

- If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is given by per annum is $\frac{P}{\left[\frac{100}{100+R}\right]^1 + \left[\frac{100}{100+R}\right]^2 + \left[\frac{100}{100+R}\right]^3 + \ldots + \left[\frac{100}{100+R}\right]^n}$

Derivation-

Principal for Rs. X due at end of first year at R% $= \frac{100X}{100+R}$

Principal for Rs. X due at end of second year at R% $= \left[\frac{100}{100+R}\right]^2 X$

……………………………………………………………………………

Principal for Rs. X due at end of nth year at R% $= \left[\frac{100}{100+R}\right]^n X$

$$\left[\frac{100}{100+R}\right]^1 X + \left[\frac{100}{100+R}\right]^2 X + \left[\frac{100}{100+R}\right]^3 X + \ldots + \left[\frac{100}{100+R}\right]^n X = P$$

$$\left\{\left[\frac{100}{100+R}\right]^1 + \left[\frac{100}{100+R}\right]^2 + \left[\frac{100}{100+R}\right]^3 + \ldots + \left[\frac{100}{100+R}\right]^n\right\} X = P$$

$$X = \frac{P}{\left[\frac{100}{100+R}\right]^1 + \left[\frac{100}{100+R}\right]^2 + \left[\frac{100}{100+R}\right]^3 + \ldots + \left[\frac{100}{100+R}\right]^n}$$

9.2 Computation of Compound Interest

Example 1

How much compound interest will be obtained on Rs.8,000 at the interest rate of 5 % p.a. after 2 years?

Solution – Given,

P = Rs. 8,000; R = 5 % p.a.; and n = 2 years

A =? and C.I. =?

We know that $A = P\left[1 + \frac{R}{100}\right]^n$

$$A = 8000 \times \left[1 + \frac{5}{100}\right]^2$$

$$A = 8000 \times \left\{\left[1 + \frac{1}{20}\right]^2\right\}$$

$$A = 8000 \times \left\{\left[\frac{20+1}{20}\right]^2\right\}$$

$$A = 8000 \times \left\{\left[\frac{21}{20}\right]^2\right\}$$

$$A = 8000 \times \{[1.05]^2\}$$

$$A = 8000 \times \{1.1025\}$$

A = Rs. 8,820

Also, C.I. = A – P

C.I. = 8,820 – 8,000 = Rs. 820

Example 2

Find compound interest on Rs. 3,000 at 5% p.a. for 2years compounded annually.

Solution – Given,

P = Rs. 3,000; R = 5% p.a.; and n = 2 years

C.I. =?

We know that $\text{C.I.} = P\left\{\left[1+\frac{R}{100}\right]^n - 1\right\}$

$\text{C.I.} = 3000 \times \left\{\left[1+\frac{5}{100}\right]^2 - 1\right\}$

$\text{C.I.} = 3000 \times \left\{\left[1+\frac{1}{20}\right]^2 - 1\right\}$

$\text{C.I.} = 3000 \times \left\{\left[\frac{20+1}{20}\right]^2 - 1\right\}$

$\text{C.I.} = 3000 \times \left\{\left[\frac{21}{20}\right]^2 - 1\right\}$

$\text{C.I.} = 3000 \times \{[1.05]^2 - 1\}$

$\text{C.I.} = 3000 \times \{1.1025 - 1\}$

$\text{C.I.} = 3000 \times \{0.1025\}$

$\text{C.I.} = 3000 \times \left\{\frac{1025}{10000}\right\}$

C.I. = Rs. 307.5

Example 3

Find the compound interest on Rs. 4,000 at 10% p.a. for 2years 6 months, compounded annually.

Solution – Given,

P = Rs. 4,000; R = 10 % p.a.; $n\frac{1}{m}$ = 2 years 6 months = $2\frac{6}{12}$ years = $2\frac{1}{2}$ years

C.I. = ?

We know that $\text{C.I.} = P\left\{\left[1+\frac{R}{100}\right]^n \times \left[1+\frac{\frac{1}{m}R}{100}\right] - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\left[1+\frac{10}{100}\right]^2 \times \left[1+\frac{\frac{1}{2}\times 10}{100}\right] - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\left[1+\frac{1}{10}\right]^2 \times \left[1+\frac{1}{20}\right] - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\left[\frac{10+1}{10}\right]^2 \times \left[\frac{20+1}{20}\right] - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\left[\frac{11}{10}\right]^2 \times \left[\frac{21}{20}\right] - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\frac{121}{100} \times \frac{21}{20} - 1\right\}$

$\text{C.I.} = 4000 \times \left\{\frac{2541}{2000} - 1\right\}$

$\text{C.I.} = 4000 \times \{1.2705 - 1\}$

$\text{C.I.} = 4000 \times \{0.2705\}$

C.I. = Rs. 1,082

Example 4

Find compound interest on Rs. 5,000 at 4% p.a. for 2 years compounded half-yearly.

Solution – Given,

P = Rs. 5,000; R = 4 % p.a.; $\frac{R}{2} = \frac{4}{2}$ % half-yearly =2% half-yearly; and n = 2 years (4 half years)

C.I. =?

We know that $C.I. = P\left\{\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n} - 1\right\}$

$$C.I. = 5000 \times \left\{\left[1 + \frac{2}{100}\right]^{4} - 1\right\}$$

$$C.I. = 5000 \times \left\{\left[1 + \frac{1}{50}\right]^{4} - 1\right\}$$

$$C.I. = 5000 \times \left\{\left[\frac{50 + 1}{50}\right]^{4} - 1\right\}$$

$$C.I. = 5000 \times \left\{\left[\frac{51}{50}\right]^{4} - 1\right\}$$

$$C.I. = 5000 \times \{[1.02]^{4} - 1\}$$

$$C.I. = 5000 \times \{1.082 - 1\}$$

$$C.I. = 5000 \times \{0.082\}$$

C.I. = Rs. 410

Example 5

Compute the compound interest on Rs. 3,000 at 20% p.a. compounded quarterly for 1 year.

Solution – Given,

P = Rs. 3,000; R = 20 % p.a. compounded quarterly; and n = 1 year (4 quarters)

C.I. =?

We know that $C.I. = P\left\{\left[1 + \frac{\frac{R}{4}}{100}\right]^{4n} - 1\right\}$

$$C.I. = 3000 \times \left\{\left[1 + \frac{\frac{20}{4}}{100}\right]^{4} - 1\right\}$$

$$C.I. = 3000 \times \left\{\left[1 + \frac{1}{20}\right]^{4} - 1\right\}$$

$$C.I. = 3000 \times \left\{\left[\frac{20+1}{20}\right]^{4} - 1\right\}$$

$$C.I. = 3000 \times \left\{\left[\frac{21}{20}\right]^{4} - 1\right\}$$

$$C.I. = 3000 \times \{[1.05]^{4} - 1\}$$

$$C.I. = 3000 \times \{1.22 - 1\}$$

$$C.I. = 3000 \times \{0.22\}$$

C.I. = Rs. 660

Example 6

Compute the compound interest on Rs. 12000 at 16% p.a. for 9 months, compounded quarterly.

Solution – Given,

P = Rs. 12000; n = 9 months = 3 quarters; and R = 16% p.a. compounded quarterly

C.I. =?

We know that C.I. = $P\left\{\left[1+\frac{\frac{R}{4}}{100}\right]^n - 1\right\}$

C.I. = $12000 \times \left\{\left[1+\frac{\frac{16}{4}}{100}\right]^3 - 1\right\}$

C.I. = $12000 \times \left\{\left[1+\frac{1}{25}\right]^3 - 1\right\}$

C.I. = $12000 \times \left\{\left[\frac{26}{25}\right]^3 - 1\right\}$

C.I. = $12000 \times \left\{[1.04]^3 - 1\right\}$

C.I. = $12000 \times \{1.12 - 1\}$

C.I. = 12000 x {0.12}

C.I. = Rs. 1,440

Example 7

Harish invests Rs. 5,000 in a bond which gives interest at 2% p.a. during the first year, 5% p.a. during the second year and 10% p.a. during the third year. Calculate compound interest.

Solution – Given,

P = Rs. 5,000; R_1 = 2% p.a.; R_2 = 5% p.a.; and R_3 = 10% p.a.

C.I. =?

We know that C.I. = $P\left\{\left[1+\frac{R_1}{100}\right] \times \left[1+\frac{R_2}{100}\right] \times \left[1+\frac{R_3}{100}\right] - 1\right\}$

C.I. = $5000 \times \left\{\left[1+\frac{2}{100}\right] \times \left[1+\frac{5}{100}\right] \times \left[1+\frac{10}{100}\right] - 1\right\}$

C.I. = $5000 \times \left\{\left[1+\frac{1}{50}\right] \times \left[1+\frac{1}{20}\right] \times \left[1+\frac{1}{10}\right] - 1\right\}$

C.I. = $5000 \times \left\{\left[\frac{50+1}{50}\right] \times \left[\frac{20+1}{20}\right] \times \left[\frac{10+1}{10}\right] - 1\right\}$

C.I. = $5000 \times \left\{\left[\frac{51}{50}\right] \times \left[\frac{21}{20}\right] \times \left[\frac{11}{10}\right] - 1\right\}$

C.I. = $5000 \times \left\{\left[\frac{11781}{10000}\right] - 1\right\}$

C.I. = $5000 \times \{1.1781 - 1\}$

C.I. = 5000 x {0.1781}

C.I. = Rs. 890.50

9.3 Computationof Principal

Example 8

Find sum of money that amounts to Rs. 5,000 after 2 years and to Rs. 10,000 after 4 years on compound interest.

Solution – Given,

A_1 = Rs. 5,000; A_2 = Rs. 10,000; n_1 = 2 years; and n_2 = 4 years

P =?

We know that A = $P\left[1+\frac{R}{100}\right]^n$ or P = $\frac{A}{\left[1+\frac{R}{100}\right]^n}$

$A_1 = P\left[1+\frac{R}{100}\right]^{n_1}$ …. (1) and $A_2 = P\left[1+\frac{R}{100}\right]^{n_2}$ …… (2)

$\frac{A_2}{A_1} = \frac{P\left[1+\frac{R}{100}\right]^{n_2}}{P\left[1+\frac{R}{100}\right]^{n_1}}$

$\frac{A_2}{A_1} = \left[1+\frac{R}{100}\right]^{n_2-n_1}$

$\frac{10,000}{5,000} = \left[1+\frac{R}{100}\right]^{4-2}$

$2 = \left[1+\frac{R}{100}\right]^{2}$

Now, A_1 = Rs. 5,000 and n_1 = 2 years

$P = \frac{5000}{\left[1+\frac{R}{100}\right]^{2}}$

$P = \frac{5,000}{2}$

P = Rs. 2,500

Example 9

Find the sum of money which will amount to Rs. 26010 in 6 months at the rate of 8% p.a. when the interest is compounded half-yearly.

Solution – Given,

A = Rs. 26,010; n = 6 months = $\frac{6}{12}$ years = $\frac{1}{2}$ years (one half); and R = 8% p.a. ($\frac{8}{2}$ = 4%)

P =?

We know that $P = \frac{A}{\left[1+\frac{\frac{R}{2}}{100}\right]^{2n}}$

$P = \frac{26010}{\left[1+\frac{4}{100}\right]^{1}}$

$P = \frac{26010}{\left[\frac{26}{25}\right]^{1}}$

$P = \frac{26010 \times 25}{26}$

P = Rs. 25009.62 (approx.)

Example 10

Find the sum of money which will amount to Rs. 26010 in 6 months at the rate of 8% p.a. when the interest is compounded quarterly.

Solution – Given,

A = Rs. 26,010; n = 6 months = $\frac{6}{12}$ years = $\frac{1}{2}$ years (2 quarters); and

R = 8% p.a. compounded quarterly ($\frac{8}{4}$ = 2% per quarter)

P =?

We know that $P = \frac{A}{\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}}$

$P = \frac{26010}{\left[1+\frac{2}{100}\right]^{2}}$

$P = \frac{26010}{\left[\frac{51}{50}\right]^{2}}$

$P = \frac{26010 \times 50 \times 50}{51 \times 51}$

P = Rs. 25,000

9.4 Computationof Rate or Rate of Interest

Example 11

At what rate percent p.a. of compound interest will Rs.16,000 amount to Rs.1, 21,500 in 5 years?

Solution – Given,

P = Rs.16,000; A = Rs.1,21,500; and n = 5 years

R =?

We know that $R = \left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100$

$R = \left\{\left[\frac{121500}{16000}\right]^{1/5} - 1\right\} \times 100$

$R = \left\{\left[\frac{243}{32}\right]^{1/5} - 1\right\} \times 100$

$R = \left\{\left[\frac{3x3x3x3x3}{2x2x2x2x2}\right]^{1/5} - 1\right\} \times 100$

$R = \left\{\left[\frac{3}{2}\right]^{(5x1)/5} - 1\right\} \times 100$

$R = \left\{\left[\frac{3}{2}\right] - 1\right\} \times 100$

$R = \left\{\frac{3-2}{2}\right\} \times 100$

$R = \left\{\frac{1}{2}\right\} \times 100$

R = 50 % p.a.

Example 12

At what rate percent p.a. of compound interest will Rs.16,000 amount to Rs. 17,640 in 2 years?

Solution – Given,

P = Rs.16,000; A = Rs.17,640; n = 2 years

R =?

We know that $R = \left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100$

$R = \left\{\left[\frac{17640}{16000}\right]^{1/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{441}{440}\right]^{1/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{21x21}{20x20}\right]^{1/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{21}{20}\right]^{(2x1)/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{21}{20}\right] - 1\right\} \times 100$

$R = \left\{\frac{21-20}{20}\right\} \times 100$

$R = \left\{\frac{1}{20}\right\} \times 100$

R = 5 % p.a.

Example 13

A sum of money Rs. 5,760 amounts to Rs. 6,250 in 1 year compounded half-yearly. Compute rate of interest p.a.

Solution – Given,

P = Rs. 5,760; A = Rs. 6,250; and n = 1 years

R =?

We know that $R = 2 \times 100 \times \left\{\left[\frac{A}{P}\right]^{1/2n} - 1\right\}$

$$R = 2 \times 100 \times \left\{\left[\frac{6250}{5760}\right]^{1/2} - 1\right\}$$

$$R = 2 \times 100 \times \left\{\left[\frac{25 \times 25}{24 \times 24}\right]^{1/2} - 1\right\}$$

$$R = 2 \times 100 \times \left\{\left[\frac{25}{24}\right]^{(2 \times 1)/2} - 1\right\}$$

$$R = 2 \times 100 \times \left\{\frac{25}{24} - 1\right\}$$

$$R = 2 \times 100 \times \left\{\frac{1}{24}\right\}$$

$$R = \frac{200}{24}$$

R = 8.33% p.a.

Example 14

A sum of money Rs. 14,641 amounts to Rs. 20,736 in 1 year compounded quarterly. Compute rate of interest p.a.

Solution – Given,

P = Rs. 14,641; A = Rs. 20,736; n = 1 years (4 quarters)

R =?

We know that $R = 4 \times 100 \times \left\{\left[\frac{A}{P}\right]^{1/4n} - 1\right\}$

$$R = 4 \times 100 \times \left\{\left[\frac{20736}{14641}\right]^{1/4} - 1\right\}$$

$$R = 4 \times 100 \times \left\{\left[\frac{12 \times 12 \times 12 \times 12}{11 \times 11 \times 11 \times 11}\right]^{1/4} - 1\right\}$$

$$R = 4 \times 100 \times \left\{\left[\frac{12}{11}\right]^{(4 \times 1)/4} - 1\right\}$$

$$R = 4 \times 100 \times \left\{\frac{12}{11} - 1\right\}$$

$$R = 4 \times 100 \times \left\{\frac{1}{11}\right\}$$

$$R = \frac{400}{11}$$

R = 36.36% p.a. (approx.)

Example 15

At what rate per cent compound interest does a sum of money become nine-fold in 2 years?

Solution – Given,

n = 9 and t = 2 years

R =?

We know that $R = 100[(n)^{1/t} - 1]$

$R = 100[(9)^{1/2} - 1]$

$R = 100[(3 \times 3)^{1/2} - 1]$

$R = 100[(3)^{(2 \times 1)/2} - 1]$

$R = 100[3 - 1]$

$R = 100[2]$

R = 200 % p.a.

Example 16

A sum of money at compound interest amounts to Rs.8,00 in three year and to Rs.1,250 in five years. Find the rate of interest p.a.

Solution – Given,

x = Rs.800; A = 3 years; y = Rs.1,250; and B = 5 years

R =?

We know that $R = \left\{\left[\frac{y}{x}\right]^{1/(B-A)} - 1\right\} \times 100$ %

$R = \left\{\left[\frac{1250}{800}\right]^{1/(5-3)} - 1\right\} \times 100$

$R = \left\{\left[\frac{25}{16}\right]^{1/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{5 \times 5}{4 \times 4}\right]^{1/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{5}{4}\right]^{(2x1)/2} - 1\right\} \times 100$

$R = \left\{\left[\frac{5}{4}\right] - 1\right\} \times 100$

$R = \left\{\frac{5 - 4}{4}\right\} \times 100$

$R = \left\{\frac{1}{4}\right\} \times 100$

R = 25 % p.a.

Example 17

A sum of money placed at C.I. doubles in 3 years. In how many years will it become four times?

Solution – Given,

n = 2, t = 3 years, and m = 2 (four times i.e., 2 x 2 = 2^2or n^m)

T =?

We know that if a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

T= m x t

T = 2 x 3

T = 6 years

Example 18

A sum of money doubles itself at C.I. in 15 years. In how many years will it become eight times?

Solution – Given,

n = 2, t = 15 years, and m = 3 (Eight times i.e., 2 x 2 x 2 = 2^3 or n^m)

T =?

We know that if a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

T = m x t

T = 3 x 15

T = 45 years

Example 19

In how many years Rs.1,00,000 will become Rs.1,33,100 at compound interest rate of 10% p.a.?

Solution – Given,

P = Rs. 1,00,000; R = 10% p.a.; and A = Rs. 1,33,100

n =?

We know that $A= P\left[1+\frac{R}{100}\right]^n$

$\frac{A}{P} = \left[1+\frac{R}{100}\right]^n$

$\frac{1,33,100}{1,00,000} = \left[1+\frac{10}{100}\right]^n$

$\frac{11 \times 11 \times 11}{10 \times 10 \times 10} = \left[1+\frac{1}{10}\right]^n$

$\left[\frac{11}{10}\right]^3 = \left[\frac{10+1}{10}\right]^n$

$\left[\frac{11}{10}\right]^3 = \left[\frac{11}{10}\right]^n$

3 = n

n = 3 years

9.5 Computationof Amount

Example 20

Mehak invested an amount of Rs. 18,000 at compound interest rate 4% p.a. for a period of 3 years. What amount will she receive at the end of 3 years?

Solution – Given,

P = Rs. 18,000; R = 4% p.a.; and n = 3 years

A =?

We know that $A= P\left[1+\frac{R}{100}\right]^n$

$A= 18000 \times \left[1+\frac{4}{100}\right]^3$

$A = 18000 \times \left[\frac{100+4}{100}\right]^3$

$A = 18000 \times \left[\frac{104}{100}\right]^3$

$$A = \frac{18000 \times 104 \times 104 \times 104}{100 \times 100 \times 100}$$

$$A = \frac{18 \times 104 \times 104 \times 104}{1000}$$

$$A = \frac{20247552}{1000}$$

A = Rs. 20,247.552

Example 21

Rohit invested an amount of Rs. 1,000 at 4% p.a. compounded half-yearly for a period of 2 years. What amount will he receive at the end of 2 years?

Solution – Given,

P = Rs. 1,000; R = 4% p.a. ($\frac{4}{2}$ % = 2 % half-year); and n = 2 years (4 half-years)

A =?

We know that $A = P\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n}$

$$A = 1000 \times \left[1 + \frac{2}{100}\right]^4$$

$$A = 1000 \times \left[\frac{100 + 2}{100}\right]^4$$

$$A = 1000 \times \left[\frac{102}{100}\right]^4$$

$$A = 1000 \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100}$$

$$A = \frac{108243216}{100000}$$

A = Rs. 1082.43 (aprrox.)

Example 22

Neha invested an amount of Rs. 24,000 at 12% p.a. compounded quarterly for a period of 1 year. What amount will she receive at the end of 1 year?

Solution – Given,

P = Rs. 24,000; R = 12% p.a. ($\frac{12}{4}$ % = 3 % per quarter); and n = 1 year (4 quarters)

A =?

We know that $A = P\left[1 + \frac{\frac{R}{4}}{100}\right]^{4n}$

$$A = 24000 \times \left[1 + \frac{3}{100}\right]^4$$

$$A = 24000 \times \left[\frac{100 + 3}{100}\right]^4$$

$$A = 24000 \times \left[\frac{103}{100}\right]^4$$

$$A = 24000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$$

$$A = \frac{2701221144}{100000}$$

A = Rs. 27012.21 (approx.)

Example 23

Find the amount on Rs. 4,000 at 10% p.a. for 2 years 6 months, compounded annually.

Solution – Given,

P = Rs. 4,000; R = 10 % p.a.; and $n\frac{1}{m}$ = 2 years 6 months = $2\frac{6}{12}$ years = $2\frac{1}{2}$ years

A = ?

We know that $A = P\left\{\left[1+\frac{R}{100}\right]^n \times \left[1+\frac{\frac{1}{m}R}{100}\right]\right\}$

$A = 4000 \times \left\{\left[1+\frac{10}{100}\right]^2 \times \left[1+\frac{\frac{1}{2}\times 10}{100}\right]\right\}$

$A = 4000 \times \left\{\left[1+\frac{1}{10}\right]^2 \times \left[1+\frac{1}{20}\right]\right\}$

$A = 4000 \times \left\{\left[\frac{10+1}{10}\right]^2 \times \left[\frac{20+1}{20}\right]\right\}$

$A = 4000 \times \left\{\left[\frac{11}{10}\right]^2 \times \left[\frac{21}{20}\right]\right\}$

$A = 4000 \times \left\{\frac{121}{100} \times \frac{21}{20}\right\}$

$A = 4000 \times \left\{\frac{2541}{2000}\right\}$

A = 4000 x {1.2705}

A = Rs. 5,082

Example 24

Amit invests Rs. 5,000 in a bond which gives interest at 2%p.a. during the first year, 5% p.a. during the second year and 10% p.a. during the third year. How much does he get at the end of the third year?

Solution – Given,

P = Rs. 5,000; R_1 = 2% p.a.; R_2 = 5% p.a.; and R_3 = 10% p.a.

A =?

We know that $A = P\left\{\left[1+\frac{R_1}{100}\right] \times \left[1+\frac{R_2}{100}\right] \times \left[1+\frac{R_3}{100}\right]\right\}$

$A = 5000 \times \left\{\left[1+\frac{2}{100}\right] \times \left[1+\frac{5}{100}\right] \times \left[1+\frac{10}{100}\right]\right\}$

$A = 5000 \times \left\{\left[1+\frac{1}{50}\right] \times \left[1+\frac{1}{20}\right] \times \left[1+\frac{1}{10}\right]\right\}$

$A = 5000 \times \left\{\left[\frac{50+1}{50}\right] \times \left[\frac{20+1}{20}\right] \times \left[\frac{10+1}{10}\right]\right\}$

$A = 5000 \times \left\{\left[\frac{51}{50}\right] \times \left[\frac{21}{20}\right] \times \left[\frac{11}{10}\right]\right\}$

$A = 5000 \times \left\{\left[\frac{11781}{10000}\right]\right\}$

A = 5000 x {1.1781}

A = Rs. 5890.50

Example 25

If a sum of Rs. 13040 is to be paid back in two equal annual instalments at $3\frac{3}{4}$ % p.a., what is the amount of each instalment?

Solution – Given,

P = Rs. 13,040; R = $3\frac{3}{4}$ % p.a. = $\frac{15}{4}$ % p.a.; and No. of instalments = 2

Amount of each instalment = ?

We know that Amount of each instalment = $\dfrac{P}{\left[\dfrac{100}{100+R}\right]^1 + \left[\dfrac{100}{100+R}\right]^2}$

$$= \frac{13040}{\left[\dfrac{100}{100+\frac{15}{4}}\right]^1 + \left[\dfrac{100}{100+\frac{15}{4}}\right]^2}$$

$$= \frac{13040}{\left[\dfrac{100}{\frac{400+15}{4}}\right]^1 + \left[\dfrac{100}{\frac{400+15}{4}}\right]^2}$$

$$= \frac{13040}{\left[\dfrac{100 \times 4}{415}\right]^1 + \left[\dfrac{100 \times 4}{415}\right]^2}$$

$$= \frac{13040}{\left[\dfrac{400}{415}\right]^1 + \left[\dfrac{400}{415}\right]^2}$$

$$= \frac{13040}{\left[\dfrac{400}{415}\right] \times \left[1 + \dfrac{400}{415}\right]}$$

$$= \frac{13040}{\left[\dfrac{400}{415}\right] \times \left[\dfrac{815}{415}\right]}$$

$$= \frac{13040 \times 415 \times 415}{400 \times 815}$$

= Rs. 6,889

9.6 Relation between Compound and Simple Interest

Example 26

What will be the difference between simple and compound interest on a sum of Rs. 30,000 put for 2 years at 5% p.a.?

Solution – Given,

P = Rs. 30,000; R = 5% p.a.; and n = 2 years

C.I. – S.I. =?

We know that C.I. – S.I. = $P\left[\dfrac{R}{100}\right]^2$

$$= 30000 \times \left[\frac{5}{100}\right]^2$$

$$= \frac{30000 \times 5 \times 5}{10000}$$

= Rs. 75

Example 27

If the difference between compound interest & simple interest on a certain sum of money for 2 years at 5% p.a. is Rs. 100, find the sum.

Solution – Given,

n = 2; R = 5% p.a.; and C.I. – S.I. = Rs. 100

P =?

We know that C.I. – S.I. = $P\left[\dfrac{R}{100}\right]^2$

$$100 = P\left[\frac{5}{100}\right]^2$$

$$100 = P\left[\frac{5 \times 5}{100 \times 100}\right]$$

$$\frac{100 \times 100 \times 100}{5 \times 5} = P$$

P = Rs. 40,000

Example 28

Find the rate percent if the difference between the compound interest and simple interest on Rs.40,000 for 2 years is Rs. 100.

Solution – Given,

P = Rs. 40,000; n = 2; C.I. – S.I. = Rs. 100

R =?

We know that C.I. – S.I. = $P\left[\frac{R}{100}\right]^2$

$100 = 40000 \times \left[\frac{R}{100}\right]^2$

$\frac{1}{400} = \left[\frac{R}{100}\right]^2$

$\frac{1}{20 \times 20} = \left[\frac{R}{100}\right]^2$

$\left[\frac{1}{20}\right]^2 = \left[\frac{R}{100}\right]^2$

$\frac{1}{20} = \frac{R}{100}$

R = 5% p.a.

Example 29

Find the rate percent if the S.I. on a certain sum of money for 2 years is Rs. 4,000 and difference between the compound interest and simple interest is Rs. 100.

Solution – Given,

n = 2; C.I. – S.I. = Rs. 100; and S.I. = Rs. 4,000

R =?

We know that C.I. – S.I. = $\frac{R \times S.I.}{2 \times 100}$

$100 = \frac{R \times 4000}{2 \times 100}$

$R = \frac{100 \times 2 \times 100}{4000}$

R = 5% p.a.

Example 30

What will be the difference between simple and compound interest on a sum of Rs. 8,000 put for 3 years at 5% p.a.?

Solution – Given,

P = 8,000; R = 5% p.a.; and n = 3 years

C.I. – S.I. =?

We know that C.I. – S.I. = $P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$

$= 8000\left\{\left[\frac{5}{100}\right]^3 + 3\left[\frac{5}{100}\right]^2\right\}$

$= 8000\left[\frac{1}{8000} + \frac{3}{400}\right]$

$= 8000\left[\frac{1+60}{8000}\right]$

= Rs. 61

Example 31

The difference between the compound interest and simple interest on a certain sum of money for 3 years at 10% p.a. is Rs. 93. Find the sum.

Solution – Given,

n = 3; R = 10% p.a.; and C.I. – S.I. = Rs. 93

P =?

We know that $\text{C.I.} - \text{S.I.} = P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$

$$93 = P\left\{\left[\frac{10}{100}\right]^3 + 3\left[\frac{10}{100}\right]^2\right\}$$

$$93 = P\left[\frac{1}{1000} + \frac{3}{100}\right]$$

$$93 = P\left[\frac{1+30}{1000}\right]$$

$$93 = P\left[\frac{1+30}{1000}\right]$$

$$93 = P\left[\frac{31}{1000}\right]$$

$$P = \frac{93 \times 1000}{31}$$

P = Rs. 3000

Example 32

The difference between compound and simple interests on a certain sum of money at the interest rate of 10% per annum for $1\frac{1}{2}$ years is Rs. 183, when the interest is compounded semi-annually. Find the sum of money.

Solution – Given,

n = 3 ($1\frac{1}{2}$ years = 3 half years); R = 10% p.a. ($\frac{R}{2} = \frac{10}{2}$ % = 5 % half yearly); and

C.I. – S.I. = Rs. 183

P =?

We know that $\text{C.I.} - \text{S.I.} = P\left\{\left[\frac{\frac{R}{2}}{100}\right]^3 + 3\left[\frac{\frac{R}{2}}{100}\right]^2\right\}$

$$183 = P\left\{\left[\frac{5}{100}\right]^3 + 3\left[\frac{5}{100}\right]^2\right\}$$

$$183 = P\left[\frac{1}{8000} + \frac{3}{400}\right]$$

$$183 = P\left[\frac{1+60}{8000}\right]$$

$$183 = P\left[\frac{1+60}{8000}\right]$$

$$183 = P\left[\frac{61}{8000}\right]$$

$$P = \frac{183 \times 8000}{61}$$

P = Rs. 24000

Summary

- Compound Interest = Amount – Principal

- When interest is compounded annually
  - Compound Interest = $P\left\{\left[1+\frac{Rate}{100}\right]^{n}-1\right\}$
  - Rate of interest (R) = $\left\{\left[\frac{A}{P}\right]^{1/n}-1\right\}$ % p.a.
- When interest is compounded Half-yearly
  - Compound Interest = $P\left\{\left[1+\frac{\frac{R}{2}}{100}\right]^{2n}-1\right\}$
- When interest is compounded Quarterly
  - Compound Interest = $P\left\{\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}-1\right\}$
- When interest is compounded annually but time is in fraction, $1\frac{1}{2}$ years.

  Compound Interest = $P\left\{\left[1+\frac{R}{100}\right]^{1} \times \left[1+\frac{\frac{1}{2}R}{100}\right]-1\right\}$
- When rates are different for different years
  - Compound Interest = $P\left\{\left[1+\frac{R_1}{100}\right] \times \left[1+\frac{R_2}{100}\right] \times \left[1+\frac{R_3}{100}\right] \times \left[1+\frac{R_4}{100}\right] \times \ldots.. -1\right\}$
- Present worth of Rs. x due n years hence
  - Present Worth = $\frac{x}{\left[1+\frac{R}{100}\right]^{n}}$
- For 2 years at R% per annum
  - In terms of P and R
    - C.I. – S.I. = $P\left[\frac{R}{100}\right]^{2}$
  - In terms of S.I. and R
    - C.I. – S.I. = $\frac{R \times S.I.}{2 \times 100}$
- For 3 years at R% per annum
  - In terms of P and R
    - C.I. – S.I. = $P\left\{\left[\frac{R}{100}\right]^{3}+3\left[\frac{R}{100}\right]^{2}\right\}$
  - In terms of S.I. and R
    - C.I. – S.I. = $\frac{S.I.}{3}\left\{\left[\frac{R}{100}\right]^{2}+3\left[\frac{R}{100}\right]\right\}$
- If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.
- If a certain sum becomes n times in t years, then the rate of compound interest is

  R = $100[(n)^{1/t}-1]$
- If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is R = $\left\{\left[\frac{y}{x}\right]^{1/(B-A)}-1\right\} \times 100$ %
- If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is

$$\frac{P}{\left[\frac{100}{100+R}\right]^{1}+\left[\frac{100}{100+R}\right]^{2}+\left[\frac{100}{100+R}\right]^{3}+\ldots+\left[\frac{100}{100+R}\right]^{n}}$$

- C.I. = A – P
- When interest is compounded annually

- C.I. $= P\left\{\left[1 + \frac{\text{Rate}}{100}\right]^n - 1\right\}$

- When interest is compounded annually but time is in fraction, $n\frac{1}{m}$ years.
  - C.I. $= P\left\{\left[1 + \frac{R}{100}\right]^n \times \left[1 + \frac{\frac{1}{m}R}{100}\right] - 1\right\}$
  - When interest is compounded Half-yearly
  - C.I. $= P\left\{\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n} - 1\right\}$
- When interest is compounded Quarterly
  - C.I. $= P\left\{\left[1 + \frac{\frac{R}{4}}{100}\right]^{4n} - 1\right\}$
- When rates are different for different years
  - C.I. $= P\left\{\left[1 + \frac{R_1}{100}\right] \times \left[1 + \frac{R_2}{100}\right] \times \left[1 + \frac{R_3}{100}\right] \times \ldots - 1\right\}$
- When interest is compounded annually
  - $A = P\left[1 + \frac{R}{100}\right]^n$
- When interest is compounded half-yearly
  - $A = P\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n}$
- When interest is compounded annually
  - $A = P\left[1 + \frac{\frac{R}{4}}{100}\right]^{4n}$
- When interest is compounded annually
  - $R = \left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100$
- When interest is compounded half-yearly
  - $R = 2 \times 100 \times \left\{\left[\frac{A}{P}\right]^{1/2n} - 1\right\}$
- When interest is compounded quarterly
  - $R = 4 \times 100 \times \left\{\left[\frac{A}{P}\right]^{1/4n} - 1\right\}$
- If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

$$T = m \times t$$

- If A, P, and R is given.

$$\frac{A}{P} = \left[1 + \frac{R}{100}\right]^n$$

- When interest is compounded annually
  - $A = P\left[1 + \frac{R}{100}\right]^n$
- When interest is compounded half-yearly
  - $A = P\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n}$
- When interest is compounded quarterly
  - $A = P\left[1 + \frac{\frac{R}{4}}{100}\right]^{4n}$

- When interest is compounded annually but time is in fraction, $n\frac{1}{m}$ years.
  - $A = P\left\{\left[1+\frac{R}{100}\right]^{n} \times \left[1+\frac{\frac{1}{m}R}{100}\right]\right\}$
- When rates are different for different years
  - $A = P\left\{\left[1+\frac{R_1}{100}\right] \times \left[1+\frac{R_2}{100}\right] \times \left[1+\frac{R_3}{100}\right]\right\}$

Amount of each instalment = $\dfrac{P}{\left[\frac{100}{100+R}\right]^{1} + \left[\frac{100}{100+R}\right]^{2}}$

- For 2 years at R% per annum
  - In terms of P and R
    - $C.I. - S.I. = P\left[\frac{R}{100}\right]^{2}$
  - In terms of S.I. and R
    - $C.I. - S.I. = \frac{R \times S.I.}{2 \times 100}$
- For 3 years at R% per annum
  - In terms of P and R
    - $C.I. - S.I. = P\left\{\left[\frac{R}{100}\right]^{3} + 3\left[\frac{R}{100}\right]^{2}\right\}$
  - In terms of S.I. and R
    - $C.I. - S.I. = \frac{S.I.}{3}\left\{\left[\frac{R}{100}\right]^{2} + 3\left[\frac{R}{100}\right]\right\}$

Keywords

Principalis the total amount of money borrowed.

Rate or Rate of Interestis the extra money paid per cent.

Timeis the duration for which the principal amount is given.

Simple Interestis the interest computed on the principal amount for the entire period it is borrowed.

Amountis theprincipal borrowed plus the interest.

Compound interest is the difference between the money borrowed and amount for that specific period.

Self Assessment

1. What would be the compound interest accrued on an amount of Rs. 8000 at the rate of 15 p.c. p.a. in 3 years?

A. Rs. 4051
B. Rs. 4167
C. Rs. 4283
D. Rs. 4325

2. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in

A. 10 years
B. 20 years
C. 12 years
D. 15 years

3. The compound interest on Rs. 2800 for 18 months at 10% p.a. is

A. Rs. 420

B. Rs. 434

C. Rs. 436.75

D. Rs. 441.35

4. Find the present worth of Rs. 9261 due 3 years, hence at 5% per annum compounded yearly.

A. Rs.7000

B. Rs.8000

C. Rs.9000

D. Rs.1000

5. A man saves ` 200 at the end of each year and lends the money at 5% compound interest. How much will it become at the end of 3 years?

A. Rs. 565.25

B. Rs. 635

C. Rs. 662.02

D. Rs. 666.50

6. A sum put out at 4% compound interest payable half-yearly amounts to `6632.55 in $1\frac{1}{2}$ years. The sum is

A. Rs. 6530

B. Rs. 6250

C. Rs. 6470

D. Rs. 6523

7. What will be the compound interest accrued on an amount of Rs.10000 @ 20 p.c. p.a. in 2 years if the interest is compounded half-yearly?

A. Rs. 4400

B. Rs. 4600

C. Rs. 4641

D. Rs. 4680

8. The difference of compound interest on `800 for 1 year at 20% per annum when compounded half-yearly and quarterly is

A. Rs. 4.40

B. Rs. 3.40

C. Rs. 6.40

D. Rs. 5.40

9. Find the compound interest on Rs. 15625 for 9 months at 16% per annum compounded quarterly.

A. Rs. 1851

B. Rs. 1941

C. Rs. 1951

D. Rs. 1961

10. Rs. 800 at 5% per annum compound interest amount to Rs. 882 in

A. 6 years

B. 2 years

C. 4 years

D. 5 years

11. The simple interest accrued on an amount of Rs. 20000 at the end of 3 years is Rs. 7200. What would be the compound interest accrued on the same amount at the same rate in the same period?

A. Rs. 8098.56

B. Rs. 8112.86

C. Rs. 8246.16

D. Rs. 8342.36

12. What will be the amount if a sum of Rs.5000 is placed at compound interest for 3 years while rate of interest for the first, second and third years is 2, 3 and 4 per cent, respectively?

A. Rs. 5643.12

B. Rs. 5463.12

C. Rs. 6413.12

D. Rs. 6553.22

13. A certain sum will amount to Rs.12,100 in 2 years at 10% per annum of compound interest, interest being compounded annually. The sum is

A. Rs. 12000

B. Rs. 6000

C. Rs. 8000

D. Rs. 10000

14. The compound interest on Rs. 2000 at 5% per annum, compounded yearly, for 2 years is

A. Rs. 315

B. Rs. 425

C. Rs. 205

D. Rs. 334

15. A man borrows money at 3% per annum interest payable yearly and lend it immediately at 5% interest (compound) payable half-yearly and thereby gains Rs. 330 at the end of the year. The sum borrowed is

A. Rs. 17,000

B. Rs.16,500

C. Rs.15,000

D. Rs.16,000

Answers for Self Assessment

l.	B	2.	D	3.	B	4.	B	5.	C
6.	B	7.	C	8.	A	9.	C	10.	B
11.	A	12.	B	13.	D	14.	C	15.	D

Review Questions

16. Find compound interest on Rs. 5000 for 2 years at 4% p.a.
17. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years?
18. Divide Rs. 8840 between A and B so that the amount received by A at the end of 8 years may be equal to the amount received by B at the end of 10 years, compound interest being at 10% per annum.
19. Find the compound interest on Rs. 1000 at 40% per annum compounded quarterly for 1 year.
20. Find the compound interest on Rs. 4000 at 24% per annum for 3 months, compounded monthly.
21. The compound interest on a sum of money for 3 years at 5% is Rs. 1324.05. What is the simple interest?
22. A sum of money placed at compound interest doubles in 3 years. In how many years will it become four times?
23. If the difference of the compound interest on a sum of money for 3years is Rs. 186. Find the sum of money if the rate of interest in both cases be 10%.
24. A sum of money at compound interest amounts to Rs. 4050 in one year and to Rs. 4723.92 in 3 years. Find the rate of interest per annum.
25. If the difference of the compound interest on a sum of money for 3years is Rs. 186. Find the sum of money if the rate of interest in both cases be 10%.

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
2. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal. S. Chand & Co Ltd. (2010).
3. Magical Book on Quicker Mathsby M Tyra, Banking Service Chronicle.
4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle.
5. Quantitative Aptitude for Competitive Examinations by Dinesh Khattar, Pearson Education (2020).

Web Links

1. https://www.hitbullseye.com/quant
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 10: Calendar

CONTENTS

Objectives

After studying this unit, you will be able to:

- define basic concepts of the calendarand odd days.
- elaborate the relationship among the day, week, month, and year.
- develop an understanding of the procedure for calculating odd days.
- calculate the number of odd days in the given period.
- solve the practical problem related to the concept of calendar.

Introduction

In various competitive exams, there are various problems related to questions on the calendar. For example, problems based on finding a day on a particular date when a date is given, the day on a particular date when the day is not given, the date on a particular day when the day is given, weekday with reference to another weekday, etc.

10.1 Basic Concept of Calendar

Calendar

It is a systematic arrangement of days, weeks, and months in a defined pattern during a year. An individual can easily recognize the required date, month, or week of a particular day with the help of a calendar.

Day

It is the fundamental unit of the calendar. There are seven days with unique names, i.e., Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. There are 24 hours in a day.

Week

The combination of seven days is called a week.

Month

The collection of a specified number of days is called a month.There are twelve months. The names of various months are January, February, March, April, May, June, July, August, September, October, November, and December.

A month contains 28 or 29 or 30 or 31 days. There are 28 days (for the non-leap year) or 29 days (for the leap year) in February. There are 30 days in April, June, September, and November. There are 31 days in January, March, May, July, August, October, and December.

Year

The total time utilized by our mother earth to complete one revolution around the center of our solar system (Sun) is known as a year.

Relation between Day, Week, Month, and Year

A day is the -

- 7^{th}part of the week.
- 28^{th}or 29^{th}or 30^{th}or 31^{st}part of a month.
- 365^{th}part of the Lunar year or 366^{th}part of the leap year.

A week is the -

- 4^{th}or 4 and $\left(\frac{1}{7}\right)^{th}$or 4 and $\left(\frac{2}{7}\right)^{th}$or 4 and $\left(\frac{3}{7}\right)^{th}$part of a month.
- 52^{nd}or 52 and $\left(\frac{1}{7}\right)^{th}$or 52 and $\left(\frac{2}{7}\right)^{th}$part of a year.

A month is the 12^{th}part of a year.

Date

It is a number/digit assigned to each day. The number assigned to a day represents that part of a month.

It is the 365^{th}part of the year for the Lunar year.

It is the 366^{th}part of the year for the leap year.

Century

It is a set/group of 100 years.

Ordinary Year

It is a year that includes 365 days or 52 full weeks and one extra day. It is not exactly divisible by 4. For example, 2010, 2011, 2021, 2022, etc.

Leap Year

It is a year that includes 366 days or 52 full weeks and two extra days. It is exactly divisible by 4. For example, 2012, 2016, 2020, 2024, etc.

Did you know?

- A century is an ordinary year if it is not divisible by 4 or 400. For example, 100, 200, 300, 500, 600, 700, and so on.
- Every 4^{th}century is a leap year or a century that is divisible by 4 or 400 is a leap year. For example, 400, 800, 1200, 1600, 2000, 2400, etc.
- The first day of a century must either be Monday, Tuesday, Thursday, or Saturday.
- The last day of a century cannot be Tuesday, Thursday, or Saturday.
- April & July for all years have the same calendar.
- January and October for non-leap years have the same calendar.

- The calendars of two different years are the same if -both years are either ordinary years or leap years, and January 21 of both years must be the same day of the week.

10.2 Calculation of Odd Days

The odd days are the number of days that are more than a week. In other words, if we divided the given number of days by seven (number of weekdays) and the digit obtained as a remainder after the result of the division is/are called odd day/days.

Therefore, the odd days do not mean odd numbers.

The concept of odd days helps a lot to compute the day of the week on the given date.

For example,

Number of days = 11

Number of weekdays = 7

Remainder = 4 (After dividing 10 by 7)

Hence, there are four odd days.

Counting of Odd Days

One ordinary year = 365 days

= (52 x 7 days + 1 day)

= (52 weeks + 1 day)

Therefore, one ordinary year has one (1) odd day.

One leap year = 366 days

= (52 x 7 days + 2 day)

= (52 weeks + 2 days)

Therefore, one leap year has two (2) odd days.

100 years = 76 ordinary years + 24 leap years

= (76 x 1 + 24 x 2) odd days

(Because one ordinary year has 1 odd day, and one leap yearhas 2 odd days.)

= (76 + 48) odd days

= 124 odd days.

= (17 x 7 days + 5 days)

= (17 x 7 weeks + 5 days)

Therefore, the number of odd days in 100 years = 5 odd days

The number of odd days in 200 years = (5 x 2)odd days

= 10 odd days

= (7 + 3) odd days

= 1 week + 3 odd days

Therefore, the number of odd days in 200 years = 3 odd days

The number of odd days in 300 years = (5 x 3)odd days

= 15 odd days

= (14 + 1) odd days

= (7 x 2 + 1) odd days

= 2 weeks + 1 odd days

Therefore, the number of odd days in 300 years = 1 odd days

The number of odd days in 400 years = (5 x 4 + 1)odd days

(Because 400th year is a leap year, so, there is +1)

= 21 odd days

= (7 x 3) odd days

= (7 x 3 + 0) odd days

= 3 weeks + 0 odd days

Therefore, the number of odd days in 400 years = 0 odd days

Hence, every 4thcentury or a century divisible by 4 or 400 has 0 odd days.

In general, the number of odd days = (7P + Q) odd days

Where Q is less than 7 and is equal to the number of odd days.

Months and Number of Odd Days

The number of odd days indifferent months of a year iscalculated below:

Month of Year	Number of Odd Days
January	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
February	= 28 days = (7 x 4 days + 0 days)= (4 weeks + 0 days) = 0 odd days (For Non-leap Year)
February	= 29 days = (7 x 4 days + 1 days)= (4 weeks + 1 days) = 1 odd days (For Leap Year)
March	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
April	= 30 days = (7 x 4 days + 2 days)= (4 weeks + 2 days) = 2 odd days
May	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
June	= 30 days = (7 x 4 days + 2 days)= (4 weeks + 2 days) = 2 odd days
July	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
August	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
September	= 30 days = (7 x 4 days + 2 days)= (4 weeks + 2 days) = 2 odd days
October	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days
November	= 30 days = (7 x 4 days + 2 days)= (4 weeks + 2 days) = 2 odd days
December	= 31 days = (7 x 4 days + 3 days)= (4 weeks + 3 days) = 3 odd days

Number of odd days in first three months of an ordinary year = (31 + 28 + 31) days

= (90) days

= (84 + 6) days

= (12 x 7 + 6) days

= 12 weeks + 6 days

= 6 odd days

Number of odd days in first six months of an ordinary year = (31 + 28 + 31 + 30 +31 +30) days

= (181) days

= (175 + 6) days

= (25 x 7 + 6) days

= 25 weeks + 6 days

= 6 odd days

Number of odd days in first nine months of an ordinary year

= (31 + 28 + 31 + 30 +31 +30 + 31 + 31 + 30) days

= (273) days

= (273 + 0) days

= (39 x 7 + 0) days

= 39 weeks + 0 days

= 0 odd days

Number of odd days in second six months of an ordinary year = (31 + 31 + 30 + 31 +30 +31) days

= (184) days

= (182 + 2) days

= (26 x 7 + 2) days

= 26 weeks + 2 days

= 2 odd days

Day of the Week andNumber of Odd Days

The number of odd days assigned to the different days of a week is mentioned below:

Day of Week	Number of Odd Day
Sunday	0
Monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

Did you know?

Number of odd days in the first three months of a leap year = 0 odd days

Number of odd days in the first six months of a leap year = 0 odd days

Number of odd days in the first nine months of a leap year = 1 odd days

Number of odd days in the second six months of a leap year = 2 odd days

10.3 Finding The Exact Day

Example 1

What was the day of the week on 15thMarch 1976?

Solution –

15thMarch 1976 = (1975 years + Period from 1.1.1976 to 15.3.1976)

Number of odd days in 1600 years = 0

Number of odd days in 300 years = 1

Number of odd days for 1900 years = 0 + 1 =1

75 years = 18 leap years + 57 ordinary years

= (18 × 2 + 57 × 1) odd days

= 93 odd days

= (7 x 13 + 2) odd days

= (13 weeks + 2 odd days)

= 2 odd days

∴ 1975 years have = (1 + 2) odd days

= 3 odd days

Period from 1.1.1976 to 15.3.1976 = January + February + 15^{th}March

= (31 + 29 +15) odd days

= 75 odd days

= (7 x 10 + 5) odd days

= (10 weeks + 5 odd days)

= 5 odd days

15^{th} March 1976 = (1975 years + Period from 1.1.1976 to 15.3.1976)

= (3 + 5) odd day

= 8 odd days

= (7 x 1 + 1) odd days

= 1 Week +1 odd day

= 1 odd day

∴ Total number of odd days till 15^{th} March 1976 = 1 odd day

Hence, the day of the week on 15^{th} March 1976 is**Monday**.

Example2

What was the day of the week on 21^{st} August 2030?

Solution –

21^{st} August 2030 = (2029 years + Period from 1.1.2030 to 21.8.2030)

Number of odd days in 2000 years = 0

29 years = 7 leap years + 22 non-leap years

= (7 × 2 + 22 × 1) odd days

= 36 odd days

= (5 weeks + 1 odd days)

= 1 odd day

∴ 2029 years have = (0 + 1) odd days

= 1 odd day

Period from 1.1.2030 to 21.8.2030 =Jan + Feb + Mar + Apr + May + Jun + Jul + 21^{st} Aug

= (31 + 28 + 31 + 30 + 31 + 30 + 31 + 21) odd days

= 233 odd days

= (33 x 7 + 2) odd days

= (33 weeks + 2 odd days)

= 2 odd days

21[st] August 2030 = (2029 years + Period from 1.1.2030 to 15.8.2030)

= (1 + 2) odd days

= 3 odd days

∴ Total number of odd days till 21[st] August 2030= 3 Odd Day

Hence, the day of the week on 21[st] August 2030 is**Wednesday**.

Example3

On what dates in May 2035 did Monday fall?

Solution –

First, we find the day on 1 May 2035.

1 May 2035 = (2034 years + Period from 1.1.2035 to 1.5.2035)

Odd days till 2000 years = 0

From 2000 to 2034 years

Odd days in 34 years = (26 ordinary years + 8 leap year)

= $(26 \times 1 + 8 \times 2)$ odd days

= 42 odd days

= 6 weeks

= 0 odd days

Odd days till 2034 = (0 + 0) odd days

= 0 odd days

Period from 1.1.2035 to 1.5.2035 = Jan + Feb + Mar + Apr + 1[st] May

= (31 + 28 + 31 + 30 + 1)odd days

= 121 odd days

= (17 weeks + 2 odd days)

= 2 odd days

Total number of odd days till 1[st] May = (0 + 2)

= 2 odd days

∴ Total number of odd days till 1[st] May 2035 = 2 odd days

1 May 2035 was Tuesday.

Monday falls on 7 May 2005

Monday falls on the **7[th], 14[th], 21[st], and 28[th] ofMay2035**.

Example4

If today is Wednesday, then what will be the day of the week after 37 days?

Solution –

Today is Wednesday

When 37 is divided by 7

$37 = 7 \times 5 + 2$

Remainder = 2

There are two odd days.

∴ Total number of odd days after 37 days = 2 odd days

Therefore, the day of the week after 37 days = Wednesday + 2 odd days

= Friday

Example 5

Are the calendars for the years 2026 and 2037 the same?

Solution –

There will be the same day on 1.1.2026 and 1.1.2037

The number of odd days between 31.12.2025 and 31.12.2036 must be equal to 0.

There are 3 leap years and 8 ordinary years in the above said period.

Number of odd days = $(3 \times 2 + 8 \times 1)$ odd days

= 14 odd days

= 2 weeks

Number of odd days = 0odd day

∴the calendar for the year 2026 and the year 2037 are the same.

Example 6

Is April and July months in a year have the same calendar?

Solution –

April + May + June = (30 + 31 + 30)odd days

= 91 odd days

= (13 x 7) odd days

= 7 weeks

Number of Odd Days = 0odd day

Therefore, April and July months in a year have the same calendar.

Summary

In short,

- One ordinary yearhas one (1) odd day.
- One leap year has two (2) odd days.
- Number of odd days in 100 years = 5 odd days
- Number of odd days in 200 years = 3 odd days
- Number of odd days in 300 years = 1 odd days
- Number of odd days in 400 years = 0 odd days
- Every 4thcentury or a century divisible by 4 or 400 has 0 odd days.
- In general, the number of odd days = (7P + Q) odd days, where Q is less than 7 and is equal to the number of odd days.
- The numbers of odd days in the different months of a year:

Month of Year	**Number of Odd Days**
January	= 3 odd days
February	= 0 odd days (For Non-leap Year)

February	= 1 odd days (For Leap Year)
March	= 3 odd days
April	= 2 odd days
May	= 3 odd days
June	= 2 odd days
July	= 3 odd days
August	= 3 odd days
September	= 2 odd days
October	= 3 odd days
November	= 2 odd days
December	= 3 odd days

- Number of odd days in the first three months of an ordinary year = 6 odd days
- Number of odd days in the first six months of an ordinary year = 6 odd days
- Number of odd days in the first nine months of an ordinary year = 0 odd days
- Number of odd days in the second six months of an ordinary year = 2 odd days
- Number of odd days in the first three months of a leap year = 0 odd days
- Number of odd days in the first six months of a leap year = 0 odd days
- Number of odd days in the first nine months of a leap year = 1 odd day
- Number of odd days in the second six months of a leap year = 2 odd days
- The number of odd days assigned to the different days of a week:

Day of Week	**Number of Odd Day**
Sunday	0
Monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

- To calculate Exact Date -

  Calculate Total Days.

  Convert Total Days into Weeks.

  If Remainder or odd days = 0 then it is Sunday.

  If Remainder is not equal to 0 or odd days =1, 2, 3, 4, 5, and 6 then it is Monday, Tuesday,Wednesday, Thursday, Friday, and Saturday respectively.

Key Words

A **calendar** is a systematic arrangement of days, weeks, and months in a defined pattern during a year.

The **day** is the fundamental unit of the calendar.

A **week** is a combination of seven days.

A**month** is a collection of a specified number of days.

The **year** is the total time utilized by our mother earth to complete one revolution around the center of our solar system (Sun).

A **date** is a number assigned to each day.

A **century** is a set of 100 years.

The ordinary year is a year that includes 365 days or 52 full weeks and one extra day.

Leap Year is a year that includes 366 days or 52 full weeks and two extra days.

The **odd days** are the number of days that are more than a week.

SelfAssessment

1. A day is the -

A. 365^{th} part of the lunar year
B. 366^{th}part of the lunar year
C. 365^{th}part of the leap year
D. 365^{th}part of the leap year or 366^{th}part of the lunar year

2. A century is an ordinary year if it is

A. divisible by 4.
B. divisible by 400.
C. not divisible by 4.
D. not divisible by 4 or 400.

3. Which of the following days will not be the first day of a century?

A. Monday
B. Tuesday
C. Friday
D. Saturday

4. Which of the following pair of days may be the last days of a century?

A. Monday and Tuesday
B. Monday and Wednesday
C. Wednesday and Thursday
D. Saturday and Sunday

5. Which of the following pair of months have the same calendars for all the years?

A. April & June
B. June & July
C. April & July
D. January & October

6. How many odd days are there in 300 years?
A. 0 odd day
B. 1 odd day
C. 3 odd days
D. 5 odd days

7. How many odd days are there in August?
A. 0 odd day
B. 1 odd day
C. 3 odd days
D. 5 odd days

8. The number of odd days in the first two months of an ordinary year is
A. 3 odd days
B. 4 odd days
C. 5 odd days
D. 6 odd days

9. The general formula to calculate the number of odd days is given by
A. 7P - Q
B. 7P x Q
C. $\frac{7P}{Q}$
D. 7P + Q

10. The number of odd days in the first four months of a leap year is
A. 2 odd days
B. 4 odd days
C. 6 odd days
D. 8 odd days

11. What was the day of the week on 25thAugust2006?
A. Monday
B. Saturday
C. Wednesday
D. Friday

12. What was the day of the week on 21stMarch 2035?
A. Sunday
B. Thursday
C. Monday
D. Wednesday

13. On what dates in February 2028 did Thursday fall?
A. 3rd, 9th, 16th, and 25th
B. 3rd, 9th, 17th, and 24th

C. 3^{rd}, 10^{th}, 17^{th}, and 24^{th}

D. 3^{rd}, 9^{th}, 17^{th}, and 23^{rd}

14. If today is Monday, then what will be the day of the week after 51 days?

A. Saturday

B. Wednesday

C. Friday

D. Thursday

15. Which of the following pair of months in an ordinary year have the same calendar?

A. March and May

B. January and March

C. February and May

D. September and November

Answers for Self Assessment

1.	A	2.	D	3.	C	4.	B	5.	C
6.	B	7.	C	8.	A	9.	D	10.	A
11.	B	12.	D	13.	C	14.	B	15.	D

Review Questions

16. What was the day of the week on 26 January2030?
17. How many days could be in 'x' weeks and 'x' days?
18. Prove that the calendar for the year 2003 will serve for the year 2014.
19. If the dates of three of the Wednesdays are even numbersfor a certain month, then, find the day of the week on $22^{nd}$of the that month.
20. If the day after tomorrow is Wednesday, what day was tomorrow's day before yesterday?
21. Prove that the calendar for the year 2009 will serve for the year 2015.
22. Suppose the day of the week on 13^{th} January 2017was Sunday. What day of the week was on January 25, 2020?
23. How many times 29^{th}day of the month occurs in 400 consecutive years?
24. The Independence Dayof India was celebrated on Sundayin the year 2021. On which day will the Independence Day of India be celebrated in the year 2036?
25. If Russia celebrated its victory day on May9, 2022 (Monday), then when will Russia celebrate its next victory day on the same day?

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
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Unit 11: Clocks

CONTENTS

Objectives

After studying this unit, you will be able to

- analyze the basics relating to concepts of clock.
- compute angle between hands of clock in different situations.
- solve problem related to angle computation between hands of clock in various competitive exams.

Introduction

In various competitive exams, there are problems related to clocks. In this chapter you will study the concept of clock and learn the way to compute angle between hands of clock in different situations.

11.1 Concepts of Clock

The circumference of a dial of a clock is divided into 60 equal parts called minute spaces.

The clock has three hands—the hour hand, the minute hand and the second hand.

The hour hand/shortest hand indicates time in hours, the minute hand/long hand indicates time in minutes, and the second hand/thin and longest hand indicates time in second.

In an hour, the hour hand covers 5-minute spaces while the minute hand covers 60 minute spaces.

Angle traced by minute hand in 60 min. = 360°

Angle traced by hour hand in 12 hrs. = 360°

The minute hand moves 12 times as fast as the hour hand.

The minute hand of clock gains 55 minutes on the hour hand of clock in 60 minutes.

In each minute, the minute hand moves through 6°.

In each minute, the minute hand gains $5\frac{1}{2}°$ than the hour hand.

Both the hands of clock coincide once in every hour.

When the hands of the clock are coincident then angle between them is 0°.

Whenboth the hands of clock coincident or opposite to each other, then they are in same straight line.

When the hands of the clock point in opposite directions then angle between them is 180°.

When the two hands of clock are 15 minute spaces apart then angle between them is90°(Right Angle).

Did you know?

The hands of a clock are twice at right angles when they are15-minute spaces apart.

The two hands of clock are 30-minute spaces apart- Opposite Directions

The clock is too fast mean the clock indicates 12:15 butcorrect time is 12 then the clock is 15 minutes too fast.

The clock is too slow mean the clock indicates 11:45 butcorrect time is 12 then the clock is 15 minutes too slow.

Both the hands of a clock are together after every $65\frac{5}{11}$ min.

Clock is running fast,if both the hands of a clock are meeting after an interval <$65\frac{5}{11}$ min.

Clock is running slow, if both the hands of a clock are meeting after an interval >$65\frac{5}{11}$ min.

Interchange of Hands

Whenever the hands of the clock interchange positions, the sum of the angles traced by hour hand and minute hand = 360°.

After x minutes

Angle traced by minute hand in x min = $(6x)°$

Angle traced by hour hand in x min = $(0.5x)°$

Therefore, $0.5x + 6x = 360$

$6.5x = 360$

$x = \frac{360}{6.5}$

$x = \frac{360 \times 10}{65}$

$x = 55\frac{5}{13}$minutes

Therefore, after every $55\frac{5}{13}$ minutes, both the hands of a clock interchange their positions.

11.2 Problems Based on Computation of Angle between Hands of Clock

Example 1

Find the angle between the minute hand and the hour hand of a clock when the time is 9.20 am.

Solution –

Angle traced by minute hand in 60 min. = 360°

Angle traced by minute hand in 1 min. $= \frac{360°}{60}$

Angle traced by it in 20 min. $= \frac{360° \times 20}{60}$

= 120°

Angle traced by the hour hand in 12 hours = 360°

Angle traced by the hour hand in 1 hours $= \frac{360°}{12}$

Angle traced by the hour hand in 9 hours 20 min. (i.e.$9\frac{20}{60}$ hr. = $9\frac{1}{3} = \frac{28}{3}$hr.) = $\frac{360° \times 28}{12 \times 3}$ = 280°

Angle traced by minute hand in 20 min. = 120 °

Angle traced by the hour hand in 9 hours 20 min. = 280°

Angle between the minute hand and the hour hand of a clock when the time is 9.20 am

= 280° - 120 ° = 160 °

Example 2

At what time between 10 and 11 o'clock will the hands of a clock be together?

Solution –

At 10 o'clock, the hour hand is at 10 and the minute hand is at 12, i.e. they are 10 min. spaces apart.

The minute hand is 10 minutes spaces ahead of the hour hand.

To be together, the minute hand must gain 50 minutes over the hour hand.

55 minutes are gained by minute hand in 60 min.

1 minute is gained by minute hand in $\frac{60}{55}$ min.

50 minutes will be gained by minute hand in $\frac{60 \times 50}{55} = 54\frac{6}{11}$ mins.

The hands will coincide at = $54\frac{6}{11}$ min. past 10.

Example 3

At what time between 3 and 4 o'clock will the hands of a clock be together ?

Solution –

At 3 o'clock, the hour hand is at 3 and the minute hand is at 12, i.e. they are 15 min. spaces apart.

The minute hand is 15 minutes spaces behind of the hour hand.

To be together, the minute hand must gain 15 minutes over the hour hand.

55 minutes are gained by minute hand in 60 min.

1 minute is gained by minute hand in = $\frac{60}{55}$ min.

15 minutes will be gained by minute hand in = $\frac{60 \times 15}{55} = 16\frac{4}{11}$ min.

The hands will coincide at $16\frac{4}{11}$ min past 3.

Example 4

At what time between 5 and 6 o'clock will the hands of a clock be at right angle?

Solution –

At 5 o'clock, the minute hand will be 25 min. spaces behind the hour hand.

When the two hands are 15 min. spaces apart, then they are at right angles.

Case I

When minute hand is 15 min. spaces behind the hour hand.

Min. hand is 15 min. behind the hour hand

Min. hand will have to gain = (25 – 15) = 10 minute spaces

55 min. spaces are gained Min. hand in 60 min.

10 min. spaces will be gained by Min. hand in $\frac{60 \times 10}{55}$ $=10\frac{10}{11}$ min.

Case II

When the minute hand is 15 min. spaces ahead of the hour hand.

When the minute hand is 15 min. spaces ahead of the hour hand .

Min. hand will have to gain = (25 + 15) = 40 minute spaces

55 min. spaces are gained Min. hand in 60 min.

40 min. spaces will be gained by Min. hand in $\frac{60 \times 40}{55}=43\frac{7}{11}$ min.

Example 5

Find at what time between 9 and 10 o'clock will the hands of a clock be in the same straight line but not together.

Solution –

At 9 o'clock, the hour hand is at 9 and the minute hand is at 12, i.e. the two hands are 15 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart.

So, the minute hand will have to gain (30 – 15) = 15 min. spaces over the hour hand.

55 minute spaces are gained in 60 min.

15 minute spaces will be gained in $\frac{60 \times 15}{55} = 16\frac{4}{11}$ min.

The hands will be in the same straight line but not together at $16\frac{4}{11}$ min. past 9.

Example 6

At what time between 8 and 9 o'clock are the hands of a clock 5 minutes apart?

Solution –

Case I -Minute hand is 3 min. spaces behind.

At 8 o'clock, the minute hand is 40 min. spaces behind the hour hand.

In this case, the minute hand has to gain (40 – 5) = 35minute spaces.

55 min. are gained in 60 min.

35 min. are gained in $\frac{60 \times 35}{55} = 38\frac{2}{11}$ min.

The hands will be 5 min. apart at $38\frac{2}{11}$ min. past 8.

Case II-Minute hand is 3 min. spaces ahead.

At 8 o'clock, the minute hand is 40 min. spaces behind the hour hand.

In this case, the minute hand must gain (40 + 5) = 45minute spaces.

55 min. are gained in 60 min.

45 min. are gained in $\frac{60 \times 45}{55}= 49\frac{1}{11}$ min.

The hands will be 5 min. apart at $49\frac{1}{11}$ min. past 8.

Example 7

Dhruvika leaves home between 8 a.m. and 9 a.m. and returns between 2 p.m. and 3 p.m. to find that the minute and hour hands have interchanged their positions. How long was Dhruvika out of the house?

Solution –

The hands will interchange positions after crossing each other 6 times i.e. they together will make (6 + 1) or 7 complete revolutions.

Since the hands interchange positions after every 55 $\frac{5}{13}$ minutes.

Required time interval = 55 $\frac{5}{13}$ x 7 = 6hr 27 $\frac{9}{13}$ min.

Example 8

A clock is set right at 9 a.m. The clock gains 20 minutes in 24 hours. What will be the true time when the clock indicates 3 p.m. on the following day?

Solution –

Time from 9 a.m. on a day to 3 p.m. on the following day = 30 hours.

24 hours 20 min. of this clock = 24 hours of the correct clock.

$\frac{73}{3}$ hrs of this clock = 24 hrs of the correct clock.

$\frac{73}{3}$ hrs of this clock = 24 hrs of the correct clock.

30 hrs of this clock = $\frac{24 \times 3 \times 30}{73}$ hrs. of the correct clock= 29 hrs 55 $\frac{48}{73}$ of correct clock.

(29hr 59 min. approx.)

The correct time is 29 hrs 55 $\frac{48}{73}$ min. (29hr 59 min. approx.) after 9 a.m. i.e. 2 hrs 55 $\frac{48}{73}$ min. (2hr 59 min. approx.) min. past 12.

Example 9

A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Solution –

Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$\frac{356}{15}$ hrs of this clock = 24 hours of correct clock

89 hrs of this clock = $\frac{24 \times 15 \times 89}{356}$hrs of correct clock = 90 hrs of correct clock.

The correct time is 11 p.m.

Example 10

The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Solution –

In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in 60 min.

60 min. are gained in $\frac{60}{55}$ x 60 =65 $\frac{5}{11}$ min.

They are together after 65 min.

Gain in 65 min. = 65 $\frac{5}{11}$ - 65= $\frac{5}{11}$ min.

Gain in 24 hours i.e. 24 x 60 min = $\frac{5 \times 24 \times 60}{11 \times 65}$ min.

The clock gains $10\frac{10}{43}$ minutes in 24 hours.

Summary

In short,

- 60 equal parts - Minute Spaces
- Hour Hand & Minute Hand/Long Hand
- In every hour
    - Both the hands coincide once.
    - Point in opposite directions once
    - Twice at right angles.
    - Whenever the hands of the clock interchange positions, the sum of the angles traced by hour hand & minute hand = 360°.

Keywords

Circumference of a dial of a clock is divided into **60 equal parts** called **minute spaces**.

Clock has **three hands**—the hour hand, the minute hand and the second hand.

Hour hand is the shortest handamong the three hands which indicates time in hours.

Minute handis thelonger hand than hour hand but shorter than second hand which indicates time in minutes.

Second hand is the thinnest and longest hand among the three hands of clock which indicates time in second.

Self Assessment

1. If a clock takes 22 seconds to strike 12, how much time will it take to strike 6?

A. 10 seconds

B. 12 seconds

C. 14 seconds

D. 16 seconds

2. At what time between 5 and 6 O'clock are the hands of a clock 3 minutes apart?

A. 24 minutes past 5

B. 22 minutes past 3

C. 26 minutes past 4

D. 28 minutes past 3

3. Find the angle between the two hands of a clock at 30 minutes past 4 O'clock.

A. 40°

B. 30°

C. 45°

D. 46°

4. A bus leaves at 12.25 noon and reaches destination at 10.45 am. The duration of the journey is

A. 22 hrs 20 min

B. 22 hrs 40 min

C. 24 hrs 20 min

D. 24 hrs 40 min

5. An accurate clock shows 8 O' clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2O' clock in the afternoon?

A. 144°

B. 150°

C. 168°

D. 180°

6. How often between 11 O'clock and 12 O'clock are the hands of a clock in integral number of minutes apart?

A. 55 times

B. 56 times

C. 58 times

D. 60 times

7. Number of times the hands of a clock are in a straight line every day is

A. 44

B. 24

C. 42

D. 22

8. At 9 :38 A.M, through how many degrees the hour hand of a clock moved since noon the previous day?

A. 323

B. 612

C. 646

D. 649

9. At 3.40, the hour hand and the minute hand of a clock form an angle of

A. 120°

B. 125°

C. 130°

D. 135°

10. The angle between the minute hand and the hour hand of a clock when the time is 8.30, is

A. 80°

B. 75°

C. 60°

D. 105°

11. A clock takes 9 seconds to strike 4 times. To strike 12 times at the same rate, the time taken is

A. 27 seconds

B. 36 seconds

C. 30 seconds

D. 33 seconds

12. How often are the hands of a clock at right angle every day?
A. 38 times
B. 44 times
C. 40 times
D. 48 times

13. A clock is set right at 5 am. The clock loses 16 minutes. in 24 hours. What will be the true time when the clock indicates 10 pm. on the 4th day?
A. 9 am
B. 11 pm
C. 11 am
D. 9 pm

14. An accurate clock shows the time as 3.00. After the hour hand has moved 135°, the time would be
A. 6.30
B. 7.30
C. 8.00
D. 9.3

15. The minute arm of a clock is 10 cm long. The number of minutes taken by the tip of the arm to travel a length of 10 cm is nearly equal to
A. 5
B. 10
C. 15
D. 20

Answers for Self Assessment

l.	A	2.	A	3.	C	4.	A	5.	D
6.	B	7.	A	8.	D	9.	C	10.	B
11.	D	12.	B	13.	B	14.	B	15.	B

Review Questions

16. At what time between 5 and 6 O'clock are the hands of a clock together?
17. At what time between 5 and 6 O'clock will the hands of a clock be at right angle?
18. Find at what time between 2 and 3 O'clock will the hands of a clock be in the same straight line but not together.
19. Find the time between 4 and 5 O'clock when the two hands of a clock are 4 minutes apart.
20. Find the angle between the two hands of a clock at 15 minutes past 4 O'clock.
21. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes. How much in day does the clock gain or lose?

22. My watch was 3 minutes slow at 5 pm on Tuesday and it was 5 minutes fast at 11 pm on Wednesday. When did it give correct time?
23. A man who went out between 3 and 4 and returned between 8 and 9, found that the hands of the watch had exactly changed places. At what time he returned?
24. The watch which gains uniformly is 2 minutes slow at noon on Sunday and is 4 minutes 48 seconds fast at 2 pm on the following Sunday. At what time the watch was correct?
25. A watch which gains uniformly is 6 minutes slow at 4 pm on a Sunday and 10 2 3 minutes fast on the following Sunday at 8 pm. During this period (Day and Time) when was the watch correct?

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
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Unit 12: Data Sufficiency and Coding Inequalities

CONTENTS

Objectives

After studying this unit, you will be able to

- solve different types of time and work-related questions.
- Efficiency impact on work.
- What happened when number of people are increased to do a work.

Introduction

Data Sufficiency is to check and test the given set of information, whether it is enough to answer a question or not. Data Sufficiency-type questions are designed to test the candidate's ability to relate given information to reach a conclusion. Moreover, the data sufficiency has wider attributes to test the candidate. On one hand, it may have problems from any of the topics of reasoning or quantitative aptitude and on the other hand, it can test a candidate's analytical skills.

Generally, a question on any of the topics such as sequence, ranking, puzzle test, coding-decoding, blood relation, and so on is followed by two, three or any number of statements. These statements may contain information to arrive at the answer. Students need to decide which of the statement(s) is/are sufficient to answer the given questions.

Types of Data Sufficiency

As of now we know what consists of the questions related to the Data Sufficiency reasoning section. Let us see the various types of Data Sufficiency one by one from below.

12.1 Blood Relation

In this type of data sufficiency, relation between 2 people will be asked and candidates need to find the statement(s) in which data is sufficient to find the relation or the correct answer.

12.2 Order & Ranking

In this type of data sufficiency, data on order of people or their ranking will be provided and candidates will need to find statement(s) in which data is sufficient to find the correct answer.

12.3 Direction & Distance

In this type of data sufficiency, data on direction of people or points and the distance traveled by a person or distance between the points will be given and candidates will need to find the statement(s) in which data is sufficient to find the direction or distance between the points.

12.4 Coding Decoding

In this type of data sufficiency, words or letters will be coded and candidates will need to find the statement(s) in which data is sufficient to find the logic applied to decide the code.

12.5 Seating Arrangement

In this type of data sufficiency, data on arrangement of people will be provided and candidates will need to find the statement(s) in which data is sufficient to find the correct answer. Seating Arrangements are of two types such as Linear Arrangement and Circular Arrangement. In Linear Arrangement, people will be arranged or seated in one or multiple rows. In Circular Arrangement, people will be arranged or seated around a circular table.

12.6 Floor Puzzle

In this type of data sufficiency, data on people living on different floors of either same or different buildings will be given and candidates will need to find the statement(s) in which data is sufficient to find the correct answer.

12.7 Scheduling

In this type of data sufficiency, data based on months, years or date will be given and candidates need to decide whether the data provided in the statements is sufficient or not.

Approach to Questions:

The following is an outline of the core approach that you should use every time you answer a Data Sufficiency question:

1. Read the question carefully and assess all information that is provided (or not provided) in the question stem. Organize this information so that you under-stand exactly what you will need to sufficiently answer the question.

2. Avoid careless assumptions. Do not assume anything that is not explicitly provided in the question stem or the statements that follow.

3. For instance, do not assume that x and y are integers unless it is explicitly given or can be deduced from the question stem or statements. Unless instructed otherwise, assume that fractions, negatives, and zero are all included in the set of potential values.

4. Make a quick judgment on which statement is easier to assess and start with that one.

5. The order in which statements are analyzed does not matter.

6. By starting with the easier statement, you simplify the decision tree and leverage easier information first.

7. To internalize the answer choices and have a system to attack them, you should use a system.

If Question is starting with is, then you have to understand that answer will be given as **YES** and **NO**.

For every question, ask yourself the following questions (if starting with statement (1)):

Is the information in statement (1) alone enough to answer the question?

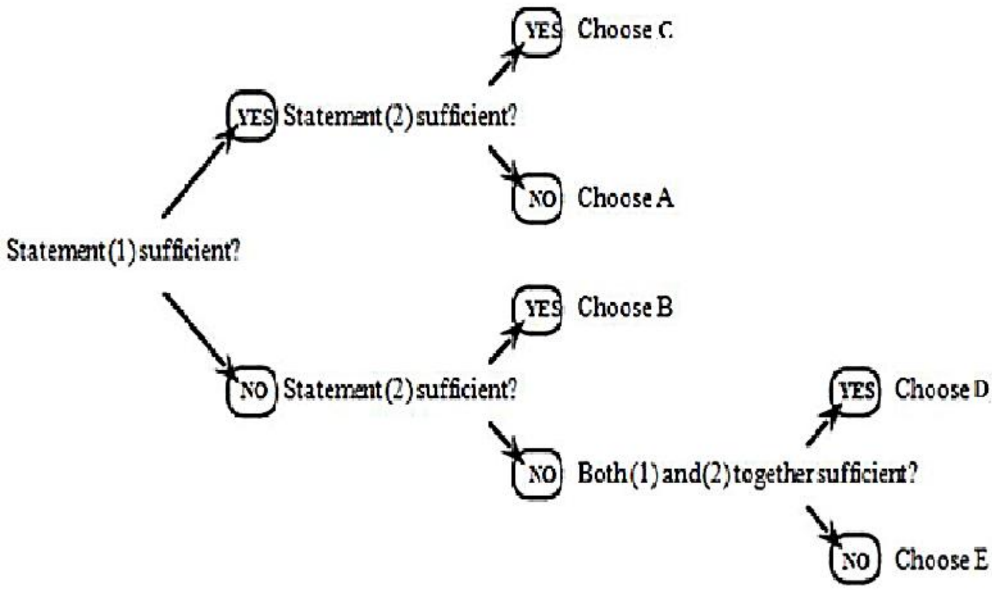
Is the information in statement (2) alone enough to answer the question?

Can I answer the question if I combine the information from statements (1) and (2)?

(Only ask this of yourself if neither statement alone was enough to answer the question.)

Data Sufficiency Decision Tree:-

Assess each statement to determine whether it is sufficient or not, and this tree will lead you to the correct answer:



Following are the possible options:

Statement (1) alone is sufficient to answer the question; Statement (2) alone is not

Statement (2) alone is sufficient to answer the question; Statement (1) alone is not

Only when considered *together* (T) do you have sufficient information to answer the question

When considered individually, *either* (E) statement provides sufficient information to answer the question

When considered alone or together, *neither* (N) statement provides sufficient information to answer the question

Example 1: Is the product of two numbers greater than 100?

A. The sum of the two numbers is greater than 50.

B. Each of the numbers is greater than 10.

Explanation: The statement A alone is not sufficient can be proved by examples. If the two numbers are 30 and 31, their sum is greater than 50 and their product is greater than 100; but if the two numbers are 50 and 1, though their sum is greater than 50, their product is only 50, and less than 100. **Statement B is sufficient**. If both of the numbers are greater than 10, then their product must be greater than 10 x 10, or greater than 100.

Example 2: Is x greater than 0?

A. x^3 is less than 0.

B. 3x = -3.

Explanation: Statement A establishes that $x^3 < 0$, so x itself must be a negative number. **Statement A alone**, therefore, is sufficient to establish that the answer to the question is "no, x is not greater than 0". Similarly, **statement B alone** also establishes that x is negative. Watch out! Some test-takers would give reasoning (incorrectly) that since the answer to the question is "no", the information is not sufficient. In fact, the information is sufficient to give a definite negative answer to the question.

Example 3: At a clothing store, Amit spent Rs. 130. How many of the articles of clothing that Amit purchased were priced at Rs. 15?

A. Amit purchased only articles costing Rs. 15 and Rs. 20.

B. Amit purchased more than two Rs. 20 articles.

Explanation: It is true that Statement A alone doesn't provide enough information to answer the question, but it does narrow the possibilities to two: two Rs. 20 articles plus six Rs. 15 articles, or five Rs. 20 articles and two Rs. 15 articles. And when the information provided in statement B is included, an answer can be obtained. Since the articles of clothing are whole articles (no fraction allowed), Amit purchased exactly two Rs. 15 articles. So, the correct answer is obtained by **combining the two statements.**

Example 4: What is the average weight of the five starting players on the defensive line of the Mammoths football team?

A. The three heaviest players on the line have an average weight of 340 pounds.

B. The two lightest players on the line weigh 275 and 290 pounds, respectively.

Explanation: To calculate the average of a group of numbers, you need just two pieces of information: the total of the numbers, and the number of numbers. In this case, the only missing piece of information is the total weight of the Mammoths defensive line. (You already know the number of numbers involved - five, since there are five players on the line.) Neither numbered statement alone gives you the players' total weight, but if you combine statements A and B, you can determine it. You'd multiply 340 by 3 (to get the combined weight of the three heaviest players) and add 275 and 290 (the weights of the two lightest players). But whatever you do, don't actually perform these steps. It's not necessary. All that matters is that you can tell that it would be theoretically possible to make these calculations and so determine the average. Hence, the answer is obtained by **combining both the statements.**

Example 5: What is x?

A. x + 2 = 4

B. $x^2 = 4$

Explanation: Statement A alone is sufficient to establish the exact value of x as 2. Statement B, however, is not sufficient. If $x^2 = 4$, x can be either + 2 or –2, and that is not sufficient to answer a question that asks "What is x?" The answer is option 1.

Example 6: What is the maximum number of cubic blocks of wood that will fit into a box?

A. The edge of each block is 2 inches long.

B. The box has the shape of a rectangular solid with inner dimensions 20 inches by 40 inches by 16 inches.

Explanation: To determine exactly how many blocks the box will hold, we need the size and shape of both the blocks and the box. Neither statement A nor statement B alone will give you all of the information you need. Statement A gives you the size and shape of the blocks while statement B gives you the size and shape of the box. Both taken together give all of the information you need. Hence, the answer is obtained by **combining both the statements.**

Example 7: In which year was Sachin born ?

I. Sachin at present is 25 years younger to his mother.

II. Sachin's brother, who was born in 1964, is 35 years younger to his mother.

Solution:

Explanation: From both the given statements, we find that Rahul is (35 - 25) = 10 years older than his brother,who was born in 1964, So. Rahul was horn in 1954

Thus, both the given statements are needed to answer the query.

Example 8: Is D brother of F ?

I. B has two sons of which F is one.

II. D's mother is married to B.

Solution

Explanation:From I, we conclude that F is the son of B.

From II, we conclude that B's wife is D's mother.

This means that D and F are the sons of B and D is the brother of F.

So, both I and II are required.

12.8 Coding Inequalities

Questions based on *Coding Inequalities* can be seen frequently in all the exams. In **Inequalities Questions**, students are provided with a statement containing different inequality symbols such as Greater than (>), Less than (<), Equals to (=), Greater than or Equals to (≥), and Less than or Equals to (≤). Students are required to decode these symbols and answer whether the given relationship of the elements in a statement is true or not. To answer such questions students are required to know the basic rules of the mathematical inequalities.

Symbols used and their implications in Coded Inequality

- **u < v** means **u is lesser than v**
- **u ≤ v** means **u is less than or equal to v**
- **u = v** means **u is equal to v**
- **u ≥ v** means **u is greater than or equal to v**
- **u > v** means **u is greater than v**
- **u ≠ v** means **u isn't equal to v**

Summary

The key concepts learnt from this Unit are: -

- We have learnt about key concepts of Data Sufficiency.
- We have learnt tricks to solve different types of Data Sufficiencyproblems
- We have learnt that data sufficiency questions are not needed to solve completely.
- We have learnt about key concept of coding inequalities.

Keywords

- Blood and Relation
- Coding Decoding
- Scheduling
- Floor puzzle
- Direction & distance
- Seating Arrangement
- Order & ranking
- Coding inequalities

SelfAssessment

Directions: Each of the questions below consists of a question and two statements numbered I and II are given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and

Give answer (A) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

Give answer (B) if the data in Statement II alone are sufficient to answer the question, while the data in

Statement I alone are not sufficient to answer the question.

Give answer (C) if the data in Statement I alone or in Statement II alone are sufficient to answer the

question.

Give answer (D) if the data in both the Statements I and II even together are not sufficient to answer the

Question.

Give answer (E) if the data in both the statements I and II even together are necessary to answer the

question.

1. Amongst A, B, C, D, E and F, each are having a different height. Who is the shortest?

I. C is shorter than only B.

II. A is taller than only D and F.

2. Point X is in which direction with respect to Y?

I . Point Z is at equal distance from both point X and point Y.

II. Walking 5 km to the East of point X and taking two consecutive right turns after walking 5 kms before each turn leads to point Y.

3. Town P is towards which direction of town T ?

I. Town T is towards South of town K, which is towards West of town P.

II. Town P is towards South of town V and towards East of town T .

4. How is “never” written is code language ?

I. “ never ever go there” is written as “ na ja ni ho” is that code language.

II. “ go there and come back” is written as “ ma ho sa ni da” is that code language

5.How is PRODUCT written in that code language?

1. In a certain code language, AIEEE is written as BJFFF.

2. In a certain code language, GYPSY is written as FXORX

6.What is Monica’s position with respect to Rahul?

1. In a row of 25 students, Monica is sitting 12th from right end of row and Rahul is sitting 20th from left end of the row.

2. Monica is 4th from right end and Rahul is 8th from left end.

7.Among A, B, C, D and E, seated in a straight line,facing North, who sits exactly in the middle of the line?

I. A sits third of left of D. B sits to the immediate right of C.

II. B sits second to right of A. E is not an immediate right of C.

8. Six people P, Q, R, S , T and U are seated around acircular table and are equidistant from each other. Who is second to the right of T ?

I. P is to the immediate left of Q and Q sits opposite R.

II. S is to the immediate left of U.

9. In a six storey building (Consisting of floors numbered 1, 2,3,4,5 and 6. The ground floor is

numbered 1, the floor above it is numbered 2 and so on) the third floor is unoccupied . The building houses different people viz. P, Q, R, S and T, each living on a different floor. On which of the floors does T live ?

I. S lives between the floors on which R and T live.

II. There are two floors between T’s floor and Q’s floor.

10. . Is S the mother of M ?

I. M is sister of Q, Q is sister of R and R is daughter of S.

II. M is daughter of L and L is sister of V.

Direction: Study the following information to answer the given questions
P$Q means P is not smaller than Q means P is neither smaller than nor equal to Q
P#Q means P is neither greater than nor equal to Q
P&Q means P is neither greater than nor smaller than Q
P*Q means P is not greater than Q

Choose correct option for each question –
A. None is true
B. Only I is true
C. Only II is true
D. Only III is true
E. Only IV is true

11. Statements: A $ M, P @ L, K # P, A $ L

Conclusions: I. K # L

II. A @ P

III.L*A

IV.M # P

12. Statements: W*N, K * V, Y @ V, W @ K

Conclusions: I. Y @ K

II. W $ N

III.W @ Y

IV.W @ V

13. Statements: T * Y, S # M, Y $ S, M @ K

Conclusions: I. K # S

II. Y @ M

III.T # M

IV.Y @ K

14. Statements: C # P, P * L, L @ E, E $ M

Conclusions: I. M # P

II. M @ L

III.P # E

IV.L # C

15. In the following question, the symbols @, CC, $, %, and # are used to illustrate the following meanings:

P $ Q means that 'P is not smaller than Q'

P # Q means 'P is not greater than Q'

P @ Q means that 'P is neither smaller than nor equal to Q'

P CC Q means that 'P is neither greater than nor equal to Q'

P % Q means 'P is neither greater than nor smaller than Q'

Statement: H % J, J CC N, N @ R

Conclusion: 1. R % J

2. H @ J

A. Only I is true.

B. Only III is true

C. Only II and III are true

D. Only I and III are true

Answers for Self Assessment

l.	D	2.	B	3.	A	4.	D	5.	E
6.	A	7.	E	8.	E	9.	DA	10.	E
11.	D	12.	B	13.	A	14.		15.	B

Review Questions

1. Who has secured less marks among P, Q, R , S & T ?

I. S has secured less marks than only R and T.

II. Q secured more marks than P.

2. On which floor is Shikha residing?

I. In a six storey building (Ground floor is parking space), Rekha is on fourth floor. Shikha likes to reside only on even numbered floors. Reema is not on the topmost floor.

II. Reema is two floors below Peter who is 3 floors above Shikha.

3 : Amit is facing which direction?

I. Shikha is facing east direction and if she turns to her right she will face Raj.

II. Amit is facing opposite direction as that of Kiran who is facing Shikha.

4: In which month is Meena's birthday?

I. Shikha remembers that Meena's birthday was 4 months ago.

II. Raj remembers that after 2 months from now, Meena's birthday will be 6 months back

5. What is the colour of white snow in a colour code?

I. Green is called Black, Black is called Blue, and Blue is called Red.

II. Red is called White and White is called Orange.

6. Who is oldest among Pete, Kevin, Joseph and Jason ?

I. Jason is older than Peter and Joseph.

II. Kevin is younger than Joseph.

7. Among five friends A, B, C, D and E sitting around a circular table and facing the centre, who is sitting to

the immediate left of A ?

I. A sits third to the right of B, D is not an immediate neighbour of B.

II. B is an immediate neighbour of C.

8. Is X the wife of Y ?

I. X's daughter M is the only sister of R. R is the son of Y.

II. The mother of Y has only one grandson R.

9. How many employees are enrolled with the company

I. The Employee Engagement survey was administered to all employees in the company .

II. A total of 346 Employee Engagement. Surveys were returned to the HR department.

10. What was the grand total of Team A ?

I. Joseph correctly remembers that Team A scored a grand total of above 85 but below 94 points.

II. Surekha correctly remembers that Team A scored a grand total of above 80 and below 87 points

11. In the following question, the symbols @, CC, $, %, and # are used to illustrate the following meanings:

P $ Q means that 'P is not smaller than Q'

P # Q means 'P is not greater than Q'

P @ Q means that 'P is neither smaller than nor equal to Q'

P CC Q means that 'P is neither greater than nor equal to Q'

P % Q means 'P is neither greater than nor smaller than Q'

2. Statements: M @ j, J $ T, T CC N

Conclusions: 1. N # J   2. T CC N

A. Only I and III are true

B. Only II is true

C. Only II and III are true

D. All I, II, and III are true

12. In the following question, the symbols @, CC, $, %, and # are used to illustrate the following meanings:

P $ Q means that 'P is not smaller than Q'

P # Q means 'P is not greater than Q'

P @ Q means that 'P is neither smaller than nor equal to Q'

P CC Q means that 'P is neither greater than nor equal to Q'

P % Q means 'P is neither greater than nor smaller than Q'

Statements: X $ Y, Y @ Z, W % Y

Conclusion: 1. X $ W   2. Z # W

A. Only II is true

B. Only I is true

C. Only III is true

D. All I, II, and III are true

Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R S Aggarwal, S Chand Publishing
2. Magical Book on Quicker Math's by M Tyra, Banking Service Chronicle

Web Links

1. 3100+ Mental Ability Reasoning Questions and Answers with Explanation (sawaal.com)
2. https://www.indiabix.com/aptitude/questions-and-answers/
3. https://www.examveda.com/mcq-question-on-arithmetic-ability/

Unit 13: Puzzle Test

CONTENTS

Objectives

After studying this unit, you will be able to

- Understand different types of Puzzles
- Understand logicto solve different arrangement-based problems
- Understand logic to solve comparison, sequential and family-based problems.

Introduction

Puzzles are raw information given for a sequence or an order of things which need to be arranged systematically, so that the sequence or order of things can be correctly depicted.

In this chapter, we deal with the questions put in the form of puzzles involving certain number of items, (may be persons or any object). You are required to analyze the given information, condense it in a suitable form and answer the questions asked.

The questions on puzzle test can be based on classification, placing arrangement, comparison, sequential order, based on family etc.

13.1 Seating/Placing Arrangements

The seating arrangement is the logical arrangement of either objects or people in a logical manner. One has to either perform the arrangement to answer the questions or decode the predefined arrangement by applying the logical analysis.

Questions based on seating arrangements involve arranging the persons or objects according to the conditions given in the question. Seating arrangement problems are based on the seating sequence pattern, direction, facing outside or inside, etc.

Seating arrangement tricks to solve the problems:

Questions on seating arrangement are generally asked in blocks of 4 – 5 questions. You are given some information and then there will 4 -5 questions based on the information. These questions have two types of information:

1. **Direct information:** This is the information which is clearly mentioned in the statement of the question. This is the information which you will use when you start solving the questions.

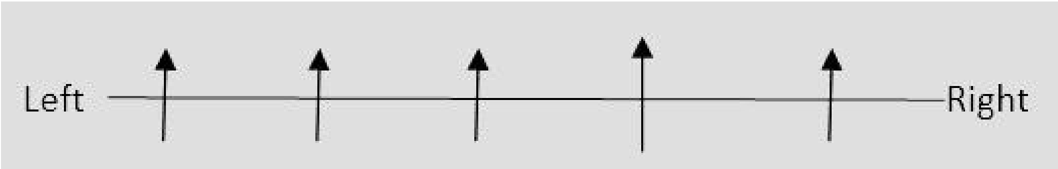
2. **Indirect information:** After filling the direct information you will look for the connection between different parts of the information. These connections form the indirect information.

While arranging the persons, the direction to which the persons are facing is very important.

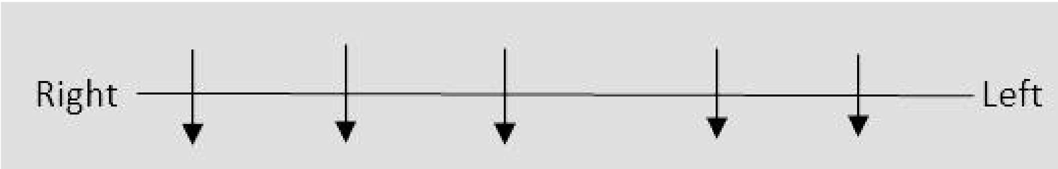
Linear arrangements

Linear Arrangement is a special type of seating Arrangement, where persons or objects are required to be placed in proper order in a straight line. candidates are required to arrange people in a row or multiple rows according to the given conditions.

Let us take the case of linear arrangements. Here if it is stated that there are five persons sitting facing North then the arrangement will be like:



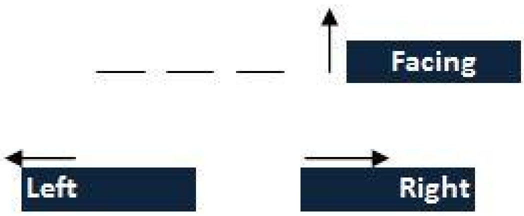
On the other hand, if these persons are sitting facing South, then the arrangement will be like:



Let us see some statements to understand this type of arrangements.

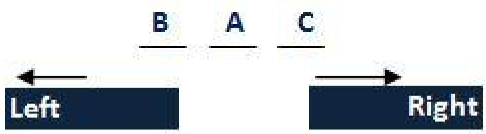
- A, B and C are standing all facing North.
- A is standing to the right of B.
- B is not standing in the middle of A and C.

If we see in the statements given above all the people are facing north. The following people will stand in the following manner:



The left and right is taken according the direction in which the person given in the question are facing. If nothing is given, we can take all people are facing north.

If we see the final arrangement of the given persons, it will be as follows:



Example:Six friends A, B, C, D, E and F are sitting in a row facing towards North. C is sitting between A and E. D is not at the end. B is sitting immediate right to E. F is not at the right end.

How many persons are there to the right D?

Solution:

In the above example, all the people are facing North. We can have the following seating arrangement:

____ ____ ____ ____ ____ ____

As C is sitting between A and E, they will be sitting in the following manner:

Case 1: A C E

Case 2: E C A

However, we still cannot fix a place for them on the above grid.

D cannot be at either of the ends. Also, B is sitting to the immediate right of E. Hence, case 2 above is discarded and we will have the following arrangement:

A C E B

In addition, F is not at the right end and D cannot be at any of the ends, we will have the following arrangement:

F D A C E B

So, on basis of the above arrangement, it can be concluded that there are four people sitting to the right of D.

Therefore, there are few important points that students should keep in mind while solving the questions based on arrangement:

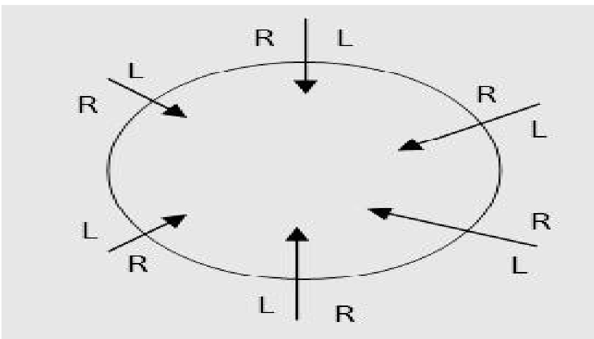
- Students should have the ability to visualize all the geometric shape of the arrangement.
- Students should have the ability to order the clues in the correct order of usage.
- Students should have the ability to understand the indirect clues.

Circular arrangements

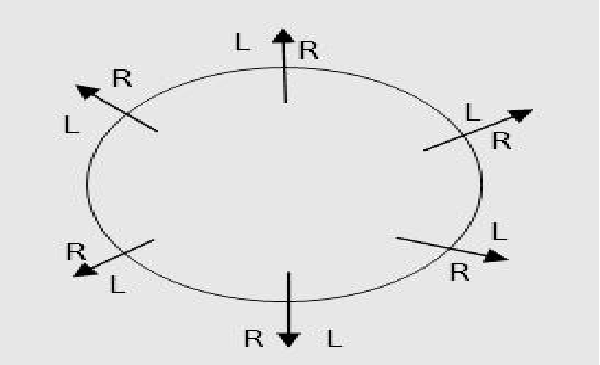
Circular Arrangement is a special type of Sitting Arrangement, where objects or persons are placed around a circle either facing the center or facing the direction opposite to center.

In this type of question some people are arranged in a circle, they may be standing or even sitting around a circular table. A student should always remember that whenever people are arranged in circles then they are always facing the center of the circle if otherwise stated.

In case of circular arrangements questions, or rectangular arrangement, the persons may be facing the centre of the circle or they may be looking away from the center. If they are looking towards the centre, then the right-hand side will be in the anticlockwise direction and left-hand side will be in the clockwise direction as shown below:



If the persons are looking away from the centre then the right hand side will be in the clockwise direction and left hand side will be in the anti-clockwise direction as shown below:



Let us understand the concept of circular arrangements through one example:

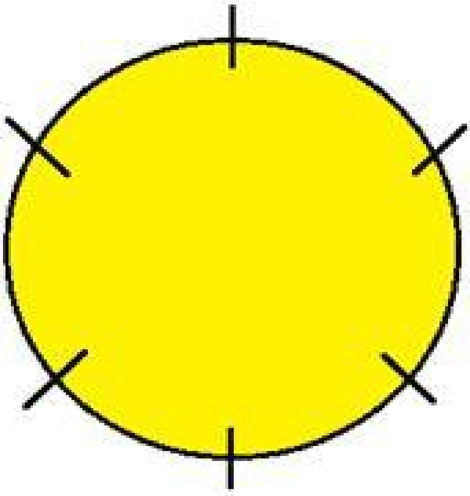
Example:

- C is sitting in-between A and F.
- B is sitting two places to the left of E.
- D is sitting two places to the right of F.

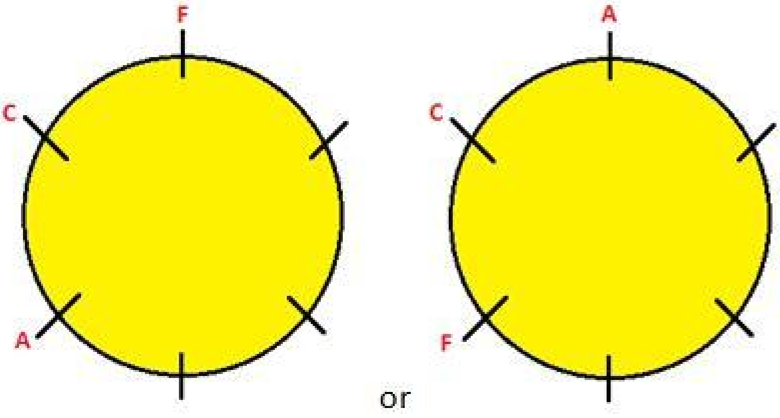
Between which two persons does D is sitting?

Solution:

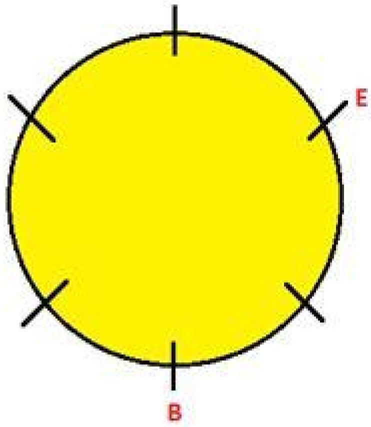
In the above example the people will be sitting in the following manner:



First statement does not give any fixed position since the order could be ACF or FCA in clockwise direction.

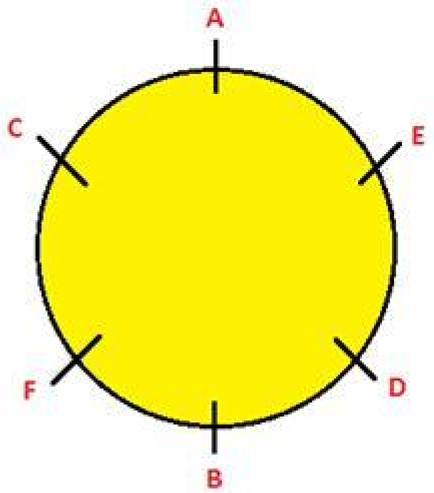


Second statement gives position of B and E.



Now C has to be in-between A and F in such an order that D is two places to the right of F.

The order in the clockwise direction has to be FCA else A will fall 2 places to the right of F which will violate the given conditions.



Thus, the final circular arrangement will be as follows:

So, D is sitting between B and E.

Therefore, there are few important points that students should keep in mind while solving the questions based on arrangement:

- Students should have the ability to visualize all the geometric shape of the arrangement.
- Students should have the ability to order the clues in the correct order of usage.
- Students should have the ability to understand the indirect clues.

13.2 Comparison Based Puzzle

In such type of questions, clues are given regarding comparisons among a set of persons or things with respect to one or more qualities. The candidate is required to analyze the whole information, form a proper ascending/descending sequence and then answer the given questions accordingly.

Example:If (i) P is taller than Q; (ii) R is shorter than P; (iii) S is taller than T but shorter than Q, then who among them is the tallest?

Solution:

In terms of height, we have: $Q < P$, $R < P$, $T < S$, $S < Q$. So, the sequence becomes: $T < S < Q < R < P$ or $T < S < R < Q < P$. Whichever may be the case, P is the tallest.

Example:Among five boys, Vineet is taller than Manick, but not as tall as Ravi. Jacob is taller than Dilip but shorter than Manick. Who is the tallest in their group?

Solution:

In terms of height, we have: Manick< Vineet, Vineet < Ravi, Dilip< Jacob, Jacob <Manick. So, the sequence becomes: Dillip< Jacob <Manick< Vineet < Ravi. Clearly, Ravi is the tallest.

13.3 Sequential Order of Things

In this type of questions, some clues are given regarding the order of occurrence of certain events. The candidate is required to analyze the given information, frame the right sequence and then answer the questions accordingly.

Example:Five boys took part in a race.

Raj finished before Mohit but behind Gaurav. Ashish finished before Sanchit but behind Mohit. Who won the race?

Solution:

Raj finished before Mohit but behind Gaurav. So, the order is Gaurav, Raj, Mohit.

Ashish finished before Sanchit but behind Mohit. So, the order is Mohit, Ashish, Sanchit.

Thus, the full order is: Gaurav, Raj, Mohit, Ashish, Sanchit.

Clearly, Gaurav won the race.

Example:Six lectures A, B, C, D, E and F are to be organized in a spam of seven days -- from Sunday to Saturday, only one lecture on each day in accordance with the following:

- A should not be organized on Thursday.
- C should be organized immediately after F.
- There should be a gap of two days between E and D.
- One day there will be no lecture (Friday is not that day), just before that day D will be organized.
- B should be organized on Tuesday and should not be followed by D.

On which day there is no lecture?

Solution:

B is organized on Tuesday. Now, D is followed by the day with no lecture. D cannot be organized on Friday because then E will be on Tuesday (there is a gap of two days between D and E). It cannot be organized on Thursday (because then, there will be no lecture Friday). B cannot be followed by D. So, D will be organized on Sunday and E on Wednesday. No lecture will be organized on Monday. A cannot be organized on Thursday. So, A will be organized on Saturday. F and C will be organized on Thursday and Friday respectively.

So, the correct order is:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
D	X	B	E	F	C	A

So, there is no lecture on Monday.

13.4 Family Based Puzzle

In this type of questions, some clues are given regarding the relationship among different members of a family together with their professions, qualities, dresses, preferences etc. The candidate is required to analyze the whole information and then answer the given questions accordingly.

Example:Prashant Arora has three children -- Sangeeta, Vimal and Ashish. Ashish married to Monika, the eldest daughter of Mr. and Mrs. Roy. The Roys married their youngest daughter to the eldest son of Mr. and Mrs. Sharma, and they had two children named Amit and Shashi. The Roys have two more children, Roshan and Vandana, both elder to Veena. Sameer and Ajay are sons of Ashish and Monika. Rashmi is the daughter of Amit. What is the surname of Rashmi?

Solution:

Rashmi is the daughter of Amit who is, therefore the eldest son of Sharmas and married to Veena, the youngest daughter of the Roys.

So, the surname of Rashmi is Sharma.

Example:

(i). P, Q, R, S, T and U are travelling in a bus.

(ii). There are two reporters, two technicians, one photographer and one writer in the group.

(iii). The photographer P is married to S who is a reporter.

(iv). The writer is married to Q who is of the same profession as that of U.

(v). P, R, Q, S are two married couples and nobody in the group has same profession.

(vi). U is brother of R.

How is R related to U?

Solution:

P is a photographer.

P is married to S. So, one couple is PS. Then, the other couple is RQ.

S is a reporter.

The writer is married to Q. So, R is the writer. Now, P, Q, R, S have different professions. So, Q is a technician and thus U is also a technician.

U is the brother of R.

We now know the professions of P, Q, R, S and U. Only T remains. Since there are two reporters in the group, so T is also a reporter.

Since the Gender of R is not given. So, R may be the brother or sister of U.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about different types of puzzles
- We have learnt logics to solve different typespuzzle-based problems

Keywords

- Seating arrangement
- Comparison
- Sequential order
- Family puzzle

Self Assessment

Direction to solve Questions (Q1-Q4):

A, B, C, D, E, F and G are sitting in a row facing North:

F is to the immediate right of E.

E is 4th to the right of G.

C is the neighbour of B and D.

Person who is third to the left of D is at one of ends.

1. Who are to the left of C?

A. Only B

B. G, B and D

C. G and B

D. D, E, F and A

2. Which of the following statement is not true?

A. E is to the immediate left of D

B. A is at one of the ends

C. G is to the immediate left of B

D. F is second to the right of D

3. Who are the neighbours of B?

A. C and D

B. C and G

C. G and F

D. C and E

4. What is the position of A?

A. Between E and D

B. Extreme left

C. Centre

D. Extreme right

Direction to solve Questions (Q5-Q7):

A, B, C, D and E are five men sitting in a line facing to south - while M, N, O, P and Q are five ladies sitting in a second line parallel to the first line and are facing to North.

B who is just next to the left of D, is opposite to Q.

C and N are diagonally opposite to each other.

E is opposite to O who is just next right of M.

P who is just to the left of Q, is opposite to D.

M is at one end of the line.

5. Who is sitting third to the right of O?

A. Q

B. N

C. M

D. Data inadequate

6. If B shifts to the place of E, E shifts to the place of Q, and Q shifts to the place of B, then who will be the second to the left of the person opposite to O?
A. Q
B. P
C. E
D. D

7. Which of the following pair is diagonally opposite to each other?
A. EQ
B. BO
C. AN
D. AM

Direction to solve Questions (Q8-Q10):

(i) Eight friends A, B, C, D, E, F, G, & H are sitting in a circle facing the

center

(ii) A, who is sitting immediately between G and C, is just opposite to F.

(iii) E, who is sitting immediately between 'H' and 'C' is second to the right of

A and second to the left of F.

(iv) D is sitting second to the left of G.

8. Who are the three friends sitting immediately to the right of B?
A. DFH
B. GAC
C. ACE
D. None of these

9. Who are the immediate neighbors of D?
A. B and F
B. F and H
C. B and G
D. None of these

10. Who is sitting directly opposite to G?
A. E
B. F
C. H
D. None of these

Direction to solve Questions (Q11-Q13):

A training college has to conduct a refresher course for teachers of seven different subjects -- Mechanics, Psychology, Philosophy, Sociology, Economics, Science and Engineering from 22nd July to 29th July.

Course should start with Psychology.

23rd July, being Sunday, should be holiday.

Science subject should be on the previous day of the Engineering subjects.

Course should end with Mechanics subject.

Philosophy should be immediately after the holiday.

There should be a gap of one day between Economics and Engineering.

The refresher course will start with which one of the following subjects?

11. The refresher course will start with which one of the following subjects?

A. Psychology

B. Mechanics

C. Philosophy

D. None of these

12. Which subject will be on Tuesday?

A. Mechanics

B. Engineering

C. Economics

D. None of these

13. Which subject precedes Mechanics?

A. Economics

B. Engineering

C. Philosophy

D. None of these

Direction to solve Questions (Q14-Q15):

(i). P, Q, R, S, T and U are six members in a family in which there are two married couples.

(ii). T, a teacher is married to the doctor who is mother of R and U.

(iii). Q, the lawyer is married to P.

(iv). P has one son and one grandson.

(v). Of the two married ladies one is a housewife.

(vi). There is also one student and one male engineer in the family.

14. How is P related to R?

A. Grandfather

B. Mother

C. Sister

D. Grandmother

15. Which of the following represents the group of females in the family?

A. PSR

B. PSU

C. QTR

D. Data inadequate

Answers for SelfAssessment

l.	C	2.	A	3.	B	4.	D	5.	B
6.	A	7.	D	8.	B	9.	A	10.	C
11.	A	12.	C	13.	D	14.	D	15.	D

Review Questions

1. A, P, R, X, S and Z are sitting in a row. S and Z are in the centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P?
2. Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary.Who is sitting immediate right to Reeta?
3. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S.Who is the fourth person to the left of Q ?
4. Five boys participated in a competition. Rohit was ranked lower than Sanjay. Vikas was ranked higher than Dinesh. Kamal`s rank was between Rohit and Vikas. Who was ranked highest?
5. Five children were administered psychological tests to know their intellectual levels. In the report, psychologists pointed out that the child A is less intelligent than the child B. The child C is less intelligent than the child D. The child B is less intelligent than the child C and child A is more intelligent than the child E. Which child is the most intelligent?

Further Reading

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, S Chand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Web Links

1. https://www.examveda.com/mcq-question-on-competitive-reasoning/
2. https://www.hitbullseye.com/Reasoning

Unit 14: Non-Verbal Reasoning

CONTENTS

Objectives

After studying this unit, you will be able to

- Understand different types of Figures based problems
- Understand logicto solve different types of Non-Verbal Reasoning
- Understand logic to solve Series, Analogy and Classification based problems.

Introduction

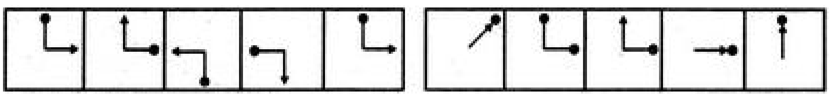
Non-verbal reasoning involves the ability to understand and analyze visual information and solve problems using visual reasoning. For example: identifying relationships, similarities and differences between shapes and patterns, recognizing visual sequences and relationships between objects, and remembering these.

14.1 Series of Figures

Series of Figures refers to a sequence of figures following some pattern. Candidate needs to find the missing term of the series by identifying the pattern involved in the formation of the series of figures.

Example:Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

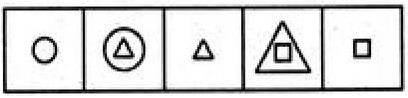
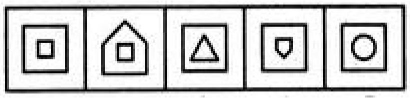
Solution:

In each step, the pin rotates 90-degree CW and the arrow rotates 90-degree ACW.

Hence, the Answer figure is fig. (3)

Example:Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

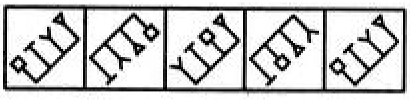
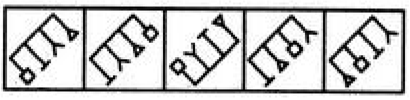
Solution:

In one step, the existing element enlarges and a new element appears inside this element. In the next step, the outer element is lost.

Hence, the Answer figure is fig. (4)

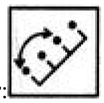
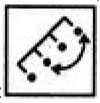
Example: Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

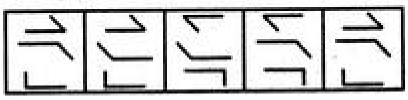
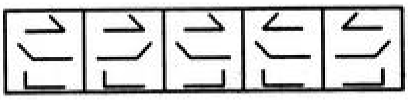
Solution:

Similar figure reappears in every second step. Each time the first figure reappears, the elements interchange positions in the order:. And, each time the second figure reappears, the elements interchange positions in the order:

Hence, the Answer figure is fig. (2)

Example:Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

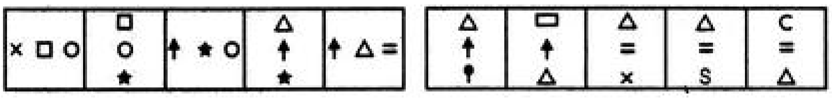
Solution:

In One step, the middle element rotates through 180 degree and in the next step, the other two elements rotate through 180 degrees. The two steps are repeated alternately.

Hence, the Answer figure is fig. (1)

Example:Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

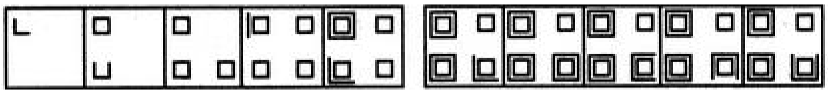
Solution:

In each step, the first element moves to the third position and gets replaced by a new element; the second and the third elements move to the first and the second positions respectively and the entire figure rotates 90-degreeCW.

Hence, the Answer figure is fig. (4)

Example:Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:



(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

Five-line segments are added in each step to complete the squares in an ACW direction.

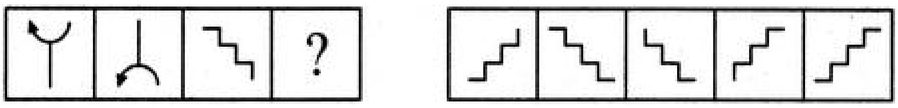
Hence, the Answer figure is fig. (5)

14.2 Analogy of Figures

In this type, two things are compared and conclusions are drawn based on their similarities. A question consists of figures related to each other based on some logic will be given, and candidates will need to find a figure analogous to those given in the question.

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:



(A) (B) (C) (D) (1) (2) (3) (4) (5)

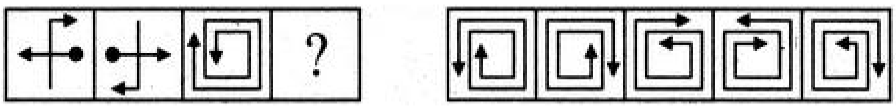
Solution:

The figure gets vertically inverted.

Hence, the Answer figure is fig. (1)

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:



(A) (B) (C) (D) (1) (2) (3) (4) (5)

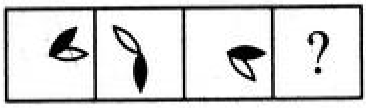
Solution:

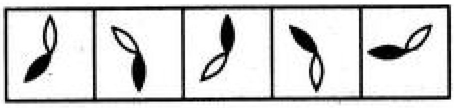
The figure gets rotated through 180- degree.

Hence, the Answer figure is fig. (2)

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

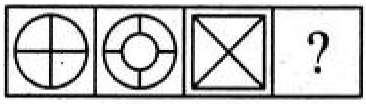
Solution:

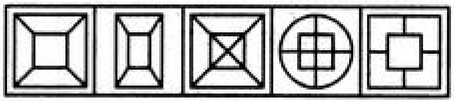
The black leaf rotates 135-degree CW and the white leaf rotates 135-degree ACW.

Hence, the Answer figure is fig. (1)

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

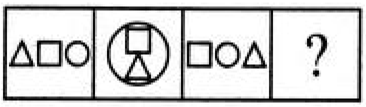
Solution:

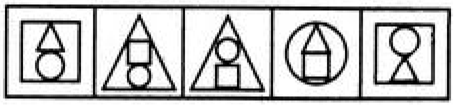
An element similar to but smaller than the outer element appears as the inner element and it hides the parts of the line segments that come under it.

Hence, the Answer figure is fig. (1)

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

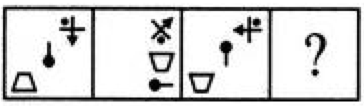
Solution:

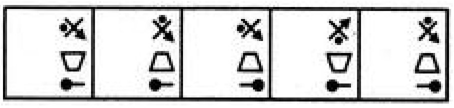
The rightmost element enlarges to become the outer element; the leftmost element becomes the inner-lower element and the middle element becomes the inner-upper element.

Hence, the Answer figure is fig. (3)

Example:Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

Solution:

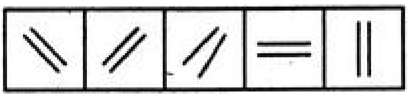
The trapezium gets vertically inverted and move to the middle right position; the pin rotates 90-degree CW and moves to the lower-right position; the third element rotates 135-degree ACW.

Hence, the Answer figure is fig. (3)

14.3 Classification of Figures

In this type, items of a given group on the basis of a certain common quality they possess and then spot the stranger or odd one out. In this case, candidates will be given a group of certain figures, out of which all except one are similar to one another in some manner.

Example:Choose the figure which is different from the rest.



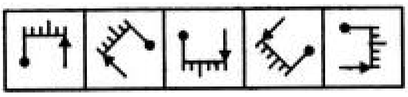
(1) (2) (3) (4) (5)

Solution:

In all the figures except fig. (3), the two-line segments are parallel to each other.

Hence, the Answer figure is fig. (3)

Example:Choose the figure which is different from the rest.



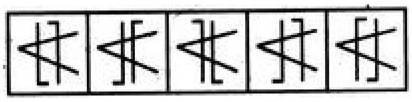
(1) (2) (3) (4) (5)

Solution:

In all the figures except fig. (1), there are two small line segments towards the pin and three small line segments towards the arrow.

Hence, the Answer figure is fig. (1)

Example:Choose the figure which is different from the rest.



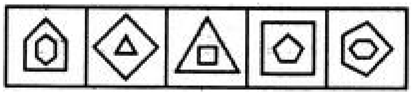
(1) (2) (3) (4) (5)

Solution:

Only in fig. (4), both the parallel lines are bent in the same direction {i.e., towards the left).

Hence, the Answer figure is fig. (4)

Example:Choose the figure which is different from the rest.



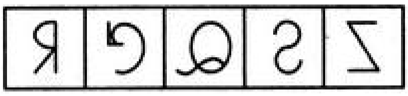
(1) (2) (3) (4) (5)

Solution:

In each one of the figures, except fig. (2), the number of sides in the inner element is one more than the number of sides in the outer element.

Hence, the Answer figure is fig. (2)

Example:Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

Solution:

Each one of the figures except fig. (2), is obtained by the lateral inversion of an English alphabet.

Hence, the Answer figure is fig. (2)

Summary

The key concepts learnt from this Unit are: -

- We have learnt about different types of Non-Verbal Figures
- We have learnt logics to solve different types of Figures based problems

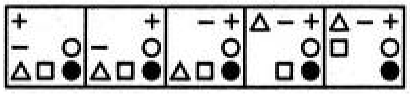
Keywords

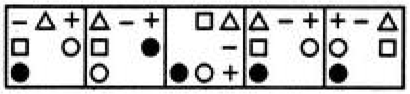
- Series
- Analogy
- Classification

Self Assessment

1. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

   Problem Figures: Answer Figures:



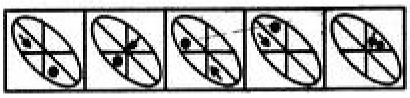


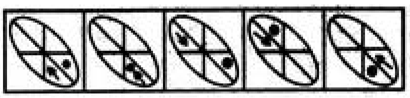
(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

   A. 1
   B. 2
   C. 3
   D. 4

2. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

   Problem Figures: Answer Figures:





(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

   A. 1

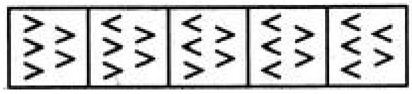
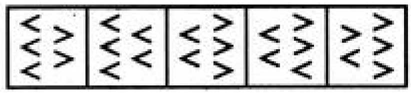
B. 2

C. 3

D. 4

3. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

A. 1

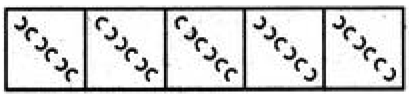
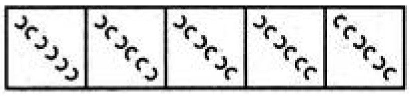
B. 2

C. 4

D. 5

4. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

A. 1

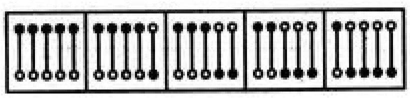
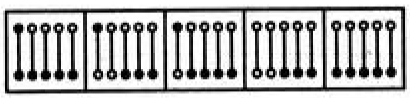
B. 2

C. 3

D. 5

5. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures: Answer Figures:

(A) (B) (C) (D) (E) (1) (2) (3) (4) (5)

A. 1

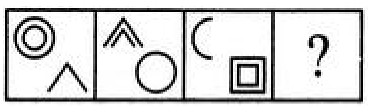
B. 2

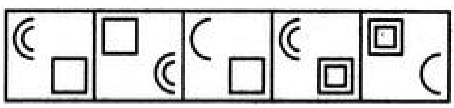
C. 4

D. 5

6. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

A. 1

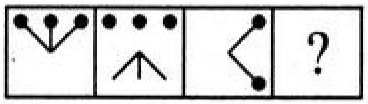
B. 2

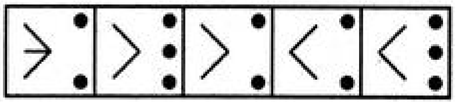
C. 3

D. 4

7. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

A. 1

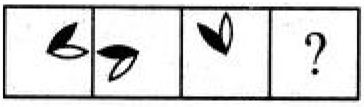
B. 2

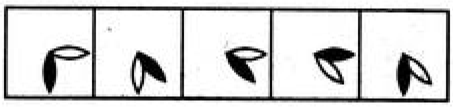
C. 3

D. 4

8. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

A. 1

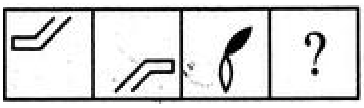
B. 2

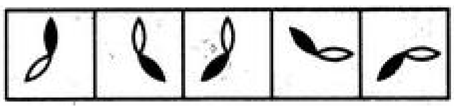
C. 5

D. 3

9. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

A. 1

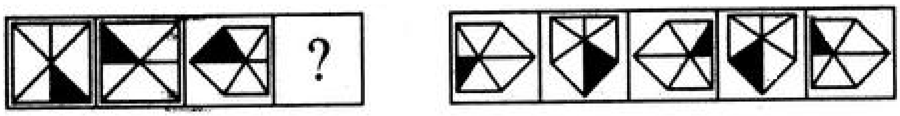
B. 2

C. 3

D. 4

10. Select a suitable figure from the Answer Figures that would replace the question mark (?).

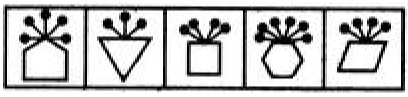
Problem Figures: Answer Figures:



(A) (B) (C) (D) (1) (2) (3) (4) (5)

A. 1
B. 2
C. 3
D. 4

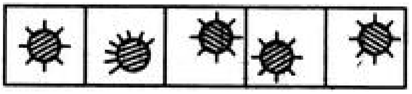
11. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

A. 1
B. 2
C. 3
D. 4

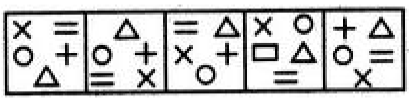
12. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

A. 1
B. 2
C. 3
D. 4

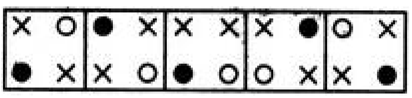
13. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

A. 1
B. 2
C. 3
D. 4

14. Choose the figure which is different from the rest.

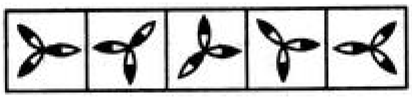


(1) (2) (3) (4) (5)

A. 1

B. 2
C. 3
D. 4

15. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

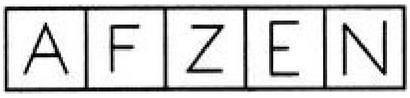
A. 1
B. 2
C. 3
D. 4

Answers for SelfAssessment

l.	D	2.	A	3.	B	4.	C	5.	D
6.	B	7.	C	8.	C	9.	C	10.	B
11.	A	12.	B	13.	D	14.	C	15.	D

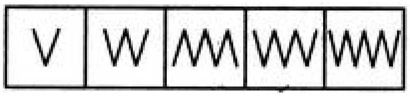
Review Questions

1. Choose the figure which is different from the rest.



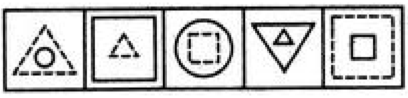
(1) (2) (3) (4) (5)

2. Choose the figure which is different from the rest.



(1) (2) (3) (4) (5)

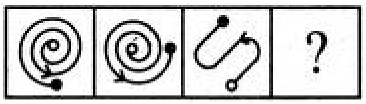
3. Choose the figure which is different from the rest.

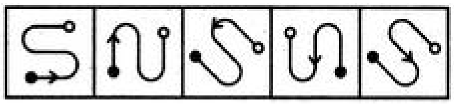


(1) (2) (3) (4) (5)

4. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:

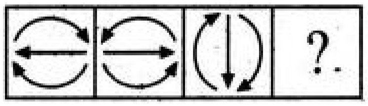


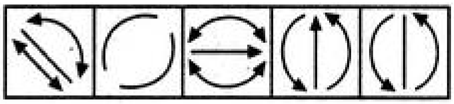


(A) (B) (C) (D) (1) (2) (3) (4) (5)

5. Select a suitable figure from the Answer Figures that would replace the question mark (?).

Problem Figures: Answer Figures:





(A) (B) (C) (D) (1) (2) (3) (4) (5)

Further Reading

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, S Chand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Web Links

1. https://www.examveda.com/mcq-question-on-competitive-reasoning/
2. https://www.hitbullseye.com/Reasoning
3. https://www.indiabix.com/non-verbal-reasoning/questions-and-answers/

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